# To Groupon or Not to Groupon: The Profitability of Deep Discounts 

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#### Abstract

We examine the profitability and implications of online discount vouchers, a relatively new marketing tool that offers consumers large discounts when they prepay for participating firms' goods and services. Within a model of repeat experience good purchase, we examine two mechanisms whereby a discount voucher service can benefit affiliated firms: price discrimination and advertising. For vouchers to provide successful price discrimination, the valuations of consumers with access to vouchers must generally be lower than those of consumers who do not have access to vouchers. Offering vouchers tends to be more profitable for firms that are patient or relatively unknown, and for firms with low marginal costs. Extensions to our model accommodate the possibilities of firm price reoptimization and multiple voucher purchases. We find potential benefits of online discount vouchers to certain firms in certain circumstances, but vouchers are likely to increase firm profits under relatively narrow conditions.


Keywords: voucher discounts, Groupon, experience goods, repeat purchase.

## 1 Introduction

In February 2013, Groupon fired its founding CEO, Andrew Mason. In some sense Mason's departure was unsurprising: Groupon had missed its own earnings projections for a second quarter in a row, and its stock was at about one quarter of its listing price. Meanwhile, most other discount voucher sites had ceased operation. Nevertheless, Mason's firing was a startling adjustment for a company and industry that, just two years ago, had been an investor favorite and a supposed sure source of easy profits.

What went wrong? In this paper, we offer a model of Groupon's core discount voucher service. Specifically, we analyze the profitability of vouchers for firms, and thus, consequently, lend insight into the sustainability of Groupon's business model.

Beginning in 2008, Groupon offered prepaid discount vouchers that soon came to include services as diverse as restaurants, skydiving, and museum visits. To consumers, discount vouchers promise substantial savings-often $50 \%$ or more. To firms, discount vouchers offer opportunities for price discrimination as well as exposure to new

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customers and online "buzz." Of the various services offering vouchers, Groupon is best-known, rising quickly from a 2008 launch to spurn a $\$ 6$ billion acquisition offer from Google in 2010, and eventually have an IPO at a $\$ 12.7$ billion valuation. Meanwhile, hundreds of websites began to offer similar discount vouchers; at one point, voucher aggregator Yipit tracked over 400 different discount voucher services. ${ }^{1}$

For firms considering whether to offer discount vouchers, the key question is whether large voucher discounts can be profitable. Voucher discounts are likely to be profitable if they predominantly attract new customers who regularly return, paying full price on future visits. But if vouchers provide discounts to many long-time customers, vouchers could sharply reduce profits. For most firms, the effect of vouchers lies between these extremes: Vouchers bring in some new customers, but also provide discounts to some regular customers. In this paper, we offer a model to explore how offer details interact with characteristics of the consumer population to shape the profitability of voucher discounts.

We illustrate two mechanisms by which a discount voucher service can benefit affiliated firms. First, discount vouchers can facilitate price discrimination, allowing firms to offer distinct prices to different consumer populations. In order for voucher offers to yield profitable price discrimination, the consumers who are offered the voucher discounts must be more price-sensitive (with regards to participating firms' goods or services) than the population as a whole. Second, discount vouchers can benefit firms through advertising, by informing consumers of a firm's existence. For these advertising effects to be important, a firm must begin with sufficiently low recognition among prospective consumers.

Our model is intentionally parsimonious, using what we believe is the minimal formalism required to properly depict discount vouchers' value propositions for firms. While many of our conclusions are intuitive from the perspective of economic theory, they are key-and we belive nonobvious-to the decisions of managers and small business owners deciding whether to engage in voucher discounting. For example, numerous business owners vocally regretted running voucher promotions-expressing surprise and disappointment that, for example, voucher customers do not often return paying full price, and that existing customers begin to use voucher discounts. (See, e.g., Dholakia (2011b,a) as well as Jessie (2010) and the comments posted therein.) Our analysis reveals and formalizes the factors that underlie these concerns. For example, we show that if vouchers are to be useful for price discrimination, then voucher-users must have systematically different (generally lower) valuations than regular customers. Our model also identifies a new reason to doubt the discount voucher business model: As services grow, they face reduced ability to provide price discrimination.

The remainder of this paper is organized as follows. We review the related literature in Section 2. We present our model of voucher discounts in Section 3, exploring price discrimination and advertising effects. In Section 4, we extend our model to consider the possibility of firms adjusting prices in connection with voucher usage and of consumers purchasing multiple vouchers. Finally, in Section 5, we discuss implications of our results for firms and voucher services. Proofs are available in our working paper (Edelman et al., 2014).

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## 2 Related Literature

A significant literature has provided empirical evidence on the impact and structure of voucher dicounting, measuring merchants' uses of vouchers and consumers' responses. The findings of this literature in part motivate our study: Dholakia (2011c) surveys businesses that offered Groupon discounts, finding that some business owners speak glowingly of Groupon, while others regret their voucher promotions. ${ }^{2}$ Byers et al. (2012a) study the dynamics of discount voucher sales as well as the impact of voucher coupon use on Yelp ratings. ${ }^{3}$

A large theoretical literature has explored the mechanisms for "group buying," also known as "social discounting," in which a certain minimum number of consumers must agree to purchase before a deal becomes effective. (See, e.g., Che and Gale (1997); Anand and Aron (2003); Jing and Xie (2011); Subramanian (2012); Meir et al. (2013); Hu et al. (2013); Liang et al. (2014); Surasvadi et al. (2014).) However, it is unclear whether group buying plays a significant role in the Groupon-style voucher model: Many voucher services do not require a minimum level of participation before offers become valid. Furthermore, Groupon has not used minimum purchase limits since May 2012 (Subramanian, 2012), and Groupon's Merchant Account Terms and Conditions oblige firms to accept whatever customer quantities Groupon provides. Moreover, Song et al. (2012) find in an empirical study of voucher offers that "it is hard to claim that the success of the daily-deal industry is due to the group shopping feature."

To the best of our knowledge, no previous study has sought to understand voucher discounting's basic value proposition for firms on a theoretical level. ${ }^{4}$ Our results indicate that voucher discounts are good fits for certain types of firms, and poor fits for others. Gupta et al. (2012) present complementary findings based on survey evidence, characterizing the types of merchants most likely to profit from offering vouchers.

Beyond the work on voucher discounts, a well-established literature explores the advertising and pricing of experience goods, i.e. goods for which some characteristics cannot be observed prior to consumption (Nelson, 1970, 1974).

The parsimonious framework of Bils (1989), upon which we base our model, shows how prices of experience goods respond to shifts in demand. Bils assumes that consumers know their conditional valuations for a firm's goods, but do not know whether that firm's goods "fit" until they have tried them. ${ }^{5}$ Analyzing overlapping consumer generations, Bils measures the tradeoff between attracting more first-time consumers and extracting surplus from returning consumers.

Meanwhile, much of the work on experience goods concerns issues of information asymmetry: If a firm's quality is unknown to consumers but known to the firm, then advertising (e.g., Nelson (1974)), introductory offers (e.g., Shapiro (1983); Milgrom and Roberts (1986)), and high initial pricing (e.g., Bagwell and Riordan (1991)), can provide signals of quality. Of this literature, the closest to our subject is the work on introductory offers. Voucher

[^1]discounts, a form of discounted initial pricing, may encourage consumers to try experience goods they otherwise would have ignored. However, we identify this effect in a setting without asymmetric information regarding firm quality-consumer heterogeneity, not information asymmetries, drives our main results. ${ }^{6}$

A substantial literature has observed that selective discounting provides opportunities for price discrimination. For example, in the setting of Varian (1980), firms engage in promotional pricing in order to attract larger market segments. ${ }^{7}$ Similar work illustrates how promotions may draw new customers (e.g.,Blattberg and Neslin (1990)), and lead those customers to become relational customers (e.g., Dholakia (2006)). These results have been found to motivate the use of coupons, especially cents-off coupons (e.g., Cremer (1984); Narasimhan (1984)). We harness the insights of the literature on sale-driven price discrimination to analyze voucher discounting-a new "sale" technology. Like the price-theoretic literature which precedes our work, we find that price discrimination depends crucially upon the presence of significant consumer heterogeneity.

Our work importantly differs from its antecedents in that the prior literature, including the articles discussed above, has considered only marginal pricing decisions. In particular, the previous work on experience goods and price discrimination does not consider deep discounts of the magnitudes now offered by voucher services.

## 3 Model

Offering a voucher through Groupon has two potential benefits to a firm: price discrimination and advertising. We present a simple model in which consumers have the opportunity to buy products from a single firm. The consumers are drawn from two populations, one of which can be targeted by voucher discount offers. First, in Section 3.1, we consider the case in which all consumers are aware of the firm and vouchers serve only to facilitate price discrimination. Then, in Section 3.2, we introduce advertising effects.

A continuum of consumers have the opportunity to buy products from a single firm. There are two periods, and the firm commits to a price $p$ for both periods. The firm's discount factor is $\delta^{f}$, while the consumers' discount factor is $\delta^{i}{ }^{8}$

Following the model of Bils (1989), we assume that each consumer $i$ has a probability $r_{i}$ that the firm's product is a "fit." Conditional on fit, the valuation of a consumer $i$ for the firm's offering is $v_{i}$. A consumer has (at most) unit demand per period.

Consumers are drawn from two populations, one of which can be targeted by voucher discount offers. ${ }^{9}$ Proportion $\lambda$ of consumers have access to vouchers, have return probability $r^{G}$, and have valuations drawn from a

[^2]distribution with cumulative distribution function $G$. Proportion $1-\lambda$ of consumers do not have access to vouchers, ${ }^{10}$ have return probability $r^{F}$, and have valuations drawn from a distribution with cumulative distribution function $F$. We assume that $F$ first-order stochastically dominates $G$, i.e. that the valuations of consumers in the $G$ population are systematically lower than those of consumers in the $F$ population. ${ }^{11}$ This is a conservative assumption: If the valuations of consumers in the $G$ population are systematically higher than those of consumers in the $F$ population, then it is only profitable to offer voucher discounts if it is profitable to offer discounts to all consumers, but this never happens if the firm's price $p$ is set optimally. The case in which all consumers come from a single population arises as the special case of our model in which $\lambda=1$.

There are two circumstances in which a purchase in the first period has positive expected value for a consumer $i$ who has valuation $v_{i}$ and return probability $r$ : First, the consumer wants to purchase if the first-period benefit is positive on its own, $r v_{i}-p \geq 0$. (For such a consumer, possible benefits later are an added bonus.) Second, the consumer wants to purchase if the expected present value of the two periods together is positive, $r v_{i}-p+\delta^{i} r\left(v_{i}-\right.$ $p) \geq 0$. So consumers $i$ with valuations $v_{i}$ and return probability $r$ purchase if

$$
\max \left\{r v_{i}-p, r v_{i}-p+\delta^{i} r\left(v_{i}-p\right)\right\} \geq 0
$$

With $\delta^{i}>0$, a consumer enjoys an informational value to visiting in the first period: If the firm's product is a fit, then the consumer knows to return. As a result, all consumers with values at least

$$
v(p ; r) \equiv \min \left\{\frac{p}{r}, \frac{1+\delta^{i} r}{\left(1+\delta^{i}\right) r} p\right\}=\frac{1+\delta^{i} r}{\left(1+\delta^{i}\right) r} p
$$

purchase in the first period.
The firm faces demand

$$
\lambda\left(1-G\left(v\left(p ; r^{G}\right)\right)\right)+(1-\lambda)\left(1-F\left(v\left(p ; r^{F}\right)\right)\right)
$$

in the first period. Fractions $r^{G}$ and $r^{F}$ of the consumers from the $G$ - and $F$-populations return in the second period. The firm maximizes profits:

$$
\pi(p) \equiv\left(\left(1+\delta^{f} r^{G}\right) \lambda\left(1-G\left(v\left(p ; r^{G}\right)\right)\right)+\left(1+\delta^{f} r^{F}\right)(1-\lambda)\left(1-F\left(v\left(p ; r^{F}\right)\right)\right)\right)(p-c)
$$

where $c$ is the firm's marginal cost. The first-order condition of the firm's optimization problem is

$$
\begin{align*}
& \lambda\left(1+\delta^{f} r^{G}\right)\left(\left(1-G\left(v^{*}\left(r^{G}\right)\right)\right)-\frac{1+\delta^{i} r^{G}}{r^{G}+\delta^{i} r^{G}}\left(p^{*}-c\right) G^{\prime}\left(v^{*}\left(r^{G}\right)\right)\right) \\
& +(1-\lambda)\left(1+\delta^{f} r^{F}\right)\left(\left(1-F\left(v^{*}\left(r^{F}\right)\right)\right)-\frac{1+\delta^{i} r^{F}}{r^{F}+\delta^{i} r^{F}}\left(p^{*}-c\right) F^{\prime}\left(v^{*}\left(r^{F}\right)\right)\right)=0 \tag{1}
\end{align*}
$$

where $p^{*}$ is the optimal price and $v^{*}(r) \equiv v\left(p^{*} ; r\right)$. We assume that the distributions of consumers are such that profits are single-peaked, so that $p^{*}$ is uniquely defined, and if the firm were able to price separately for each population, those prices would also be uniquely defined.

[^3]
### 3.1 Discount Vouchers

In this section we examine the price discrimination aspects of discount vouchers. That is, we assume all consumers are familiar with the firm, so that vouchers offer no advertising effect. We thus examine directly the question of how effectively discount vouchers can allow a firm to price discriminate among subsets of consumers.

After setting its optimal price $p^{*}$, a firm has the opportunity to offer a discount voucher. ${ }^{12}$ We assume that the firm offers vouchers to fraction $\gamma$ of the consumers in the $G$-population. ${ }^{13}$ Through the voucher service, the $G$ population consumers have the opportunity to purchase from the firm at a discounted price $\alpha p$ in the first period. If $\alpha>r^{G}$, then all the $G$-population voucher-users who purchase in the first period return if the firm is a fit. The minimal valuation of $G$-population consumers who purchase using vouchers is

$$
\tilde{v}\left(p ; r^{G}\right) \equiv \frac{\alpha+\delta^{i} r^{G}}{\left(1+\delta^{i}\right) r^{G}} p<v\left(p ; r^{G}\right) .
$$

If $\alpha<r^{G}$, then some $G$-population consumers $i$ with $v_{i}<p$ use the voucher to purchase in the first period but do not return (even if the firm is a fit). These are the $G$-population consumers $i$ for whom

$$
\begin{equation*}
r^{G} v_{i}-\alpha p>0 \quad \text { and } \quad v_{i}<p . \tag{2}
\end{equation*}
$$

The voucher service retains fraction $1-\beta$ of the voucher-discounted sales price, so the firm's revenue from each sale with discount voucher is $\alpha \beta p^{*}$. Define the voucher-user cutoff valuation $\tilde{v}^{*}(r) \equiv \tilde{v}\left(p^{*} ; r\right)$ (if $\left.\alpha>r^{G}\right)$ and $\hat{v}^{*}(r) \equiv \frac{\alpha}{r} p^{*}$ (if $\alpha<r^{G}$ ). Then offering discount vouchers yields the following change in firm profit:

$$
\Delta \pi\left(p^{*}\right)=\gamma \lambda \begin{cases}\left(\left(1+\delta^{f} r^{G}\right)\left(p^{*}-c\right)\left(G\left(v^{*}\left(r^{G}\right)\right)-G\left(\tilde{v}^{*}\left(r^{G}\right)\right)\right)-(1-\alpha \beta) p^{*}\left(1-G\left(\tilde{v}^{*}\left(r^{G}\right)\right)\right)\right) & \alpha>r^{G},  \tag{3}\\ \left(p^{*}-c\right)\left(G\left(v^{*}\left(r^{G}\right)\right)-G\left(\hat{v}^{*}\left(r^{G}\right)\right)+\delta^{f} r^{G}\left(G\left(v^{*}\left(r^{G}\right)\right)-G\left(p^{*}\right)\right)\right)-(1-\alpha \beta) p^{*}\left(1-G\left(\hat{v}^{*}\left(r^{G}\right)\right)\right) & \alpha<r^{G} .\end{cases}
$$

The first term in each component of (3) represents the increased profits from the additional consumers who purchase thanks to the voucher, $G\left(v^{*}\left(r^{G}\right)\right)-G\left(\tilde{v}^{*}\left(r^{G}\right)\right) .{ }^{14}$ The second term represents profits lost by lowering the first-period per-unit revenue from $p^{*}$ to $\alpha \beta p^{*}$ for consumers with vouchers. Note that $\gamma$ affects the magnitude of the change in profits, but not the sign. ${ }^{15}$ The following result is therefore immediate.

Proposition 1 If it is profitable to offer the voucher discount to one consumer (randomly drawn) from the G population, then it is more profitable to offer the voucher discount to all G-population consumers.

When $\alpha=1$, introducing the discount voucher does not affect consumers' purchase decisions ( $v^{*}=\tilde{v}$ ), as consumers are offered no actual "discount." Meanwhile, if consumers who use discount vouchers are ex ante identical to other consumers $\left(F(v)=G(v)\right.$ for all $v$ and $\left.r^{G}=r^{F}\right)$, then the optimality of the price $p^{*}$ implies that using

[^4]vouchers (i.e. choosing $\alpha<1$ ) is unprofitable. However, if $G$ is to the left of $F$ and $r^{G}=r^{F}$ (voucher-users have lower valuations) or if $r^{G}>r^{F}$ and $F(v)=G(v)$ (voucher-users are more likely to return), then there is some $\underline{\alpha}<1$ such that offering a discounted price of $\alpha p \geq \underline{\alpha} p$ is profitable if $\beta$ is sufficiently close to 1 . This is because the firm wants to offer a lower price to consumers from the $G$ population. Setting $\alpha<1$ brings the firm's price to the $G$-population consumers closer to the optimal price for that population. We formalize these observations in the following proposition.

Proposition 2 If $r^{G}=r^{F}$ and $F(v)=G(v)$ for all $v$ and $\min \{\alpha, \beta\}<1$, then discount vouchers yield a decrease in profit. By contrast,

- if $r^{G}=r^{F}$ and $F(v)<G(v)$ for all $v \in V$,
- if $r^{G}>r^{F}$ and $F(v)=G(v)$ for all $v \in V$, or
- if $r^{G}>r^{F}$ and $F(v)<G(v)$ for all $v \in V$,
then there is some $\underline{\alpha}<1$ such that offering a discount voucher with discounted price of $\alpha p^{*} \geq \underline{\alpha} p^{*}$ is profitable so long as $\beta$ is sufficiently close to 1 .

Proposition 2 shows that, for the purpose of price discrimination, a firm would prefer to target discounts to consumers with systematically lower valuations. Indeed, if the sole benefit of vouchers is price discrimination, then discounting cannot be profitable if everyone in the market has access to vouchers. Thus, the easier it is for consumers to join voucher services, the greater firms' risk of "cannibalization" in which full-price customers become voucherusing customers, making vouchers ineffective.

The conditions of Proposition 2 are sufficient for the profitability of vouchers but not necessary. Indeed, whenever $F, G, r^{F}$, and $r^{G}$ are such that the firm's optimal price for consumers from the $G$ population is lower than the firm's optimal price for the combined distribution of consumers, there exist some $\alpha<1$ and $\beta<1$ for which the discount voucher is profitable for the firm. ${ }^{16}$

Patience (high $\delta^{f}$ ) can have both direct effects and indirect effects on voucher profitability. The direct effect is unambiguously positive for the profitability of vouchers: With greater patience, a firm better appreciates the future profits from returning voucher consumers. The indirect effect operates through the effect of $\delta^{f}$ on $p^{*}$ (patience affecting the pre-voucher optimal price). Since the effect of $p^{*}$ on voucher profitability depends on distributions $F$ and $G$, in general it is not possible to sign the indirect effect or the total effect. ${ }^{17}$

Some firms have observed that consumers with vouchers are less likely to return paying full price. This will occur if $G$-population consumers are ex ante less likely to return than $F$-population consumers ( $r^{G}<r^{F}$ ). However, the effect can arise even when the two populations have the same ex ante probability of return $\left(r^{G}=r^{F}\right)$. If the price is too discounted ( $\alpha$ is below $r^{G}$ ), then some consumers will purchase vouchers even though their valuations conditional on a fit $\left(v_{i}\right)$ are below the undiscounted price. Such consumers will not return even if the firm's product

[^5]is a fit. Fraction $r^{G}$ of regular consumers return, but a smaller fraction
$$
r^{G} \frac{1-G\left(p^{*}\right)}{1-G\left(\frac{\alpha}{r^{G}} p^{*}\right)}
$$
of voucher consumers return. The larger the discount, the larger the fraction of voucher consumers who do not return even if the firm is a fit.

Note that we assume that all vouchers sold are redeemed. In practice, some vouchers are not used. In the United States, Groupon pays firms the standard revenue share $\alpha \beta p^{*}$ even if a consumer does not redeem a voucher. If a voucher is unredeemed in the first period, then the firm avoids the cost of serving that consumer. However, a consumer who fails to redeem a voucher will not experience the firm's product and thus will not return in the second period. Thus, non-redemption has both positive and negative effects on firm profits. Formally, if fraction $\omega$ of voucher-users fail to redeem their vouchers, then the change in profits (3) is replaced with

$$
\Delta^{\omega} \pi\left(p^{*}\right)=\Delta \pi\left(p^{*}\right)+\omega \gamma \lambda \begin{cases}\left(1-G\left(\tilde{v}^{*}\left(r^{G}\right)\right)\right)\left(c-r^{G} \boldsymbol{\delta}^{f}\left(p^{*}-c\right)\right) & \alpha>r^{G}  \tag{4}\\ \left(1-G\left(\tilde{v}^{*}\left(r^{G}\right)\right)\right) c-r^{G} \boldsymbol{\delta}^{f}\left(1-G\left(p^{*}\right)\right)\left(p^{*}-c\right) & \alpha<r^{G}\end{cases}
$$

Equation (4) confirms that, in general, the impact of nonredemption on firm profits is unclear. For example, if $\alpha>r^{G}$, nonredemption increases profits if and only if $p^{*}<\left(1+\frac{1}{r^{G} \delta^{f}}\right) c$.

### 3.2 Advertising Effects

In addition to offering consumers discounts, vouchers may also serve to inform consumers about the existence and details of affiliated firms.

We suppose that fraction $\kappa$ of consumers know about the firm, but fraction $1-\kappa$ do not. The uninformed consumers may have high valuations for the firm's product (conditional on learning about the firm), but they cannot purchase without first being informed of the firm's existence.

We assume that the probability of knowing about a firm is constant across consumers in the $G$-population (independent of valuations). ${ }^{18} \mathrm{~A}$ firm's profits absent an online voucher are given by $\pi_{\kappa}\left(p^{*}\right) \equiv \kappa \pi\left(p^{*}\right)$; the optimal price $p^{*}$ is unchanged from that defined by (1). If a firm offers a voucher, then all consumers offered the voucher learn of the firm's existence. The change in profits from offering vouchers is

$$
\Delta \pi_{\kappa}\left(p^{*}\right)=\kappa \Delta \pi\left(p^{*}\right)+\gamma \lambda(1-\kappa) \begin{cases}\left(\left(\alpha \beta p^{*}-c\right)+\delta^{f} r^{G}\left(p^{*}-c\right)\right)\left(1-G\left(\tilde{v}\left(r^{G}\right)\right)\right) & \alpha>r^{G}  \tag{5}\\ \left(\alpha \beta p^{*}-c\right)\left(1-G\left(\hat{v}\left(r^{G}\right)\right)\right)+\delta^{f} r^{G}\left(p^{*}-c\right)\left(1-G\left(p^{*}\right)\right) & \alpha<r^{G}\end{cases}
$$

Because uninformed consumers do not purchase without the voucher, no profits are foregone by offering those consumers discounted pricing. Thus, so long as the voucher discount is not so big that forgone profits in the first

[^6]period outweigh gains in the second period, the second term in (5) is positive. A sufficient (but not necessary) condition for this to occur is that the firm's per-voucher revenue is larger than its marginal cost:
\[

$$
\begin{equation*}
\alpha \beta p^{*}>c \tag{6}
\end{equation*}
$$

\]

Condition (6) is not sufficient for overall voucher profitability-it does not guarantee that $\Delta \pi\left(p^{*}\right)$ is positive. However, with advertising, as long as condition (6) holds, vouchers are always profitable for "sufficiently unknown" firms, as there is always some $\kappa>0$ such that the second term of (5) dominates the first.

Proposition 3 Suppose that the firm's post-discount revenue per-sale exceeds the firm's marginal cost, i.e. that $\alpha \beta p^{*} \geq c$. Then, there exists some $\bar{\kappa}>0$ such that offering vouchers is profitable whenever $\kappa<\bar{\kappa}$.

Unlike in the pure price discrimination case, when vouchers provide advertising benefits, the effect of the difference between the distributions of valuations on the profitability of vouchers is ambiguous. For a fixed $p^{*}$, the additional profits from advertising are higher when the valuations drawn from $G$ are higher (the difference between distributions is smaller) or when $r^{G}$ is higher, so voucher-users are more likely to return. However, changing $G$ and $r^{G}$ changes $p^{*}$, so that the net effect the change depends on the shapes of $F$ and $G$.

In general, the advertising effect will be most beneficial for newer, less well-known firms. However, those firms are often unable to afford short-term costs in order to obtain the long-term gains from repeat customers.

## 4 Extensions

### 4.1 If the Firm May Re-optimize Its Price

Even with the opportunity of price re-optimization, discount vouchers are not profitable when the two populations are identical $\left(r^{G}=r^{F}\right.$ and $F(v)=G(v)$ for all $v$ ). However, if the distributions are different, firms can increase the profitability of discount vouchers by re-optimizing their baseline prices to take into account the effects of vouchers (although vouchers still may not be profitable). The firm will set a new price $p^{* *}$ to maximize

$$
\begin{aligned}
& \gamma \lambda\left(\alpha \beta p^{* *}-c+\delta^{f} r^{G}\left(p^{* *}-c\right)\right)\left(1-G\left(\tilde{v}\left(p^{* *} ; r^{G}\right)\right)\right) \\
& \quad+\left(1+\delta^{f} r^{G}\right)\left(\lambda(1-\gamma)\left(1-G\left(v\left(p^{* *} ; r^{G}\right)\right)\right)+(1-\lambda)\left(1+\delta^{f} r^{F}\right)\left(1-F\left(v\left(p^{* *} ; r^{G}\right)\right)\right)\right) .
\end{aligned}
$$

Pricing at $p^{* *}$ raises the profits from a discount voucher promotion. The prospect of adjusting prices places new importance on $\gamma$ : The sign of the change in profits now depends on $\gamma$ because the proportion of consumers receiving the discounted price affects the firm's re-optimization. When $\gamma$ is low, the firm is limited in the extent to which it can raise the undiscounted price faced by consumers from the $F$ population because many $G$-population consumers face the same price.

### 4.2 If Consumers May Purchase Multiple Vouchers

So far we have assumed that each consumer can purchase at most one voucher, a restriction which lets us model the discounted price $\alpha p^{*}$ as available only in the first period. However, if consumers can buy multiple discount vouchers, then they can enjoy the discounted price in both periods.

Allowing consumers to buy multiple discount vouchers has ambiguous effects on firm profits. Suppose all discount vouchers must be purchased in the first period, consistent with Groupon's standard voucher format, which only makes a given voucher available for purchase on specific days. Suppose also that the discount is sufficiently large that some consumers choose to buy a second voucher $\left(\alpha<\delta^{i} r^{G}\right)$. A consumer with valuation $v_{i}$ only finds it profitable to buy a second voucher if the expected value of second-period consumption is greater than the voucher cost, that is, if

$$
\delta^{i} r^{G} v_{i} \geq \alpha p^{*}
$$

The change in profits from allowing the purchase of a second voucher (relative to offering only one voucher) is

$$
\begin{equation*}
\Delta \pi_{2}=\left(\alpha \beta p^{*}-r^{G} \delta^{f} c\right)\left(G\left(p^{*}\right)-G\left(\frac{\alpha}{r^{G} \delta^{i}} p^{*}\right)\right)+p^{*}\left(\alpha \beta-\delta^{f} r^{G}\right)\left(1-G\left(p^{*}\right)\right) \tag{7}
\end{equation*}
$$

The first term of (7) reflects the consumers who are induced to return, with a discount, by the availability of the second voucher. The firm is paid the discounted price for these consumers and must pay future costs for those that return. Some of these consumers learn in their first visit that the firm is not a fit; these consumers do not use their second voucher, while the firm retains the prepayments for their second vouchers (as discussed in Section 3.1).

The second term of (7) reflects the consumers who buy a second voucher, although they would have returned paying full price if they had not been able to purchase a second voucher. The firm would prefer not to offer a second voucher to these consumers. ${ }^{19}$

So long as the firm's price, net of discounts, is above marginal cost, the consumers from the first term of (7) yield a positive contribution towards firm profit. But inframarginal consumers in the second term of (7) produce a clear reduction in profit.

## 5 Discussion

### 5.1 Implications for Firms

Our results offer practical advice for firms considering whether to offer discount vouchers. Our discussion in Section 4.2 indicates that a firm might want to disallow purchase of multiple discount vouchers. However, as Friedman and Resnick (2001) point out, firms face substantial practical difficulties in implementing this restriction, as consumers can often create multiple accounts to purchase multiple vouchers.

[^7]In the short run, most firms cannot adjust prices in anticipation of consumers using discount vouchers. For example, restaurants-a major sector of discoun voucher providers-incur (literal) menu costs in adjusting prices. Meanwhile, the need to attract non-discounted purchases tempers a firm's incentive to increase prices: As a firm increases its posted prices, non-voucher-users are less likely to find the firm attractive, and voucher-users are less likely to return. A firm might seek to increase prices for voucher-users only, for example via a special menu for customers with vouchers. But voucher services are skeptical of this approach, perceiving that it disappoints customers who see the resulting "discount" as a bad deal. In a widely-publicized example, Groupon in 2011 offered a FTD Flowers "discount" that required customers to purchase goods through a special link with prices above FTD's ordinary prices. Facing customer complaints and media inquiry, Groupon provided refunds (Arrington, 2011).

Given the importance of attracting new customers versus the cost of offering discounts to existing customers, firms seek to assess whether their voucher-using customers have previously visited and paid full price. Firms should also measure how many voucher-using customers later come back without vouchers. In principle, credit card systems could track this information with little harm to customer privacy or data security. But in practice, most firms currently lack the tools or expertise to run such analyses. Notably, Groupon in 2012 acquired a point-of-sale system providing computerized tracking of customer visits and orders; Groupon quickly updated this system to accommodate tracking of Groupon vouchers as well as new and returning customers. Nonetheless, in many businesses with cash payment and without customer reservations, long-term tracking remains difficult.

### 5.2 The Future of Voucher Services

We first presented a version of this work in October 2010. At that time there was widespread optimism for the future of discount voucher services-literally hundreds of "Groupon clones" had attempted to copy Groupon's approach, and bankers and investors stood ready to support Groupon's $\$ 12.7$ billion valuation. By 2013 the shine had worn off: At its lowest, Groupon traded at one tenth of its IPO valuation, and firms that previously offered discount vouchers had widely turned away from this marketing strategy.

Analysts have offered a variety of potential explanations for the decline of online discount vouchers. Many flagged implementation problems like floods of emails in customers' inboxes and general "voucher fatigue" (Ewoldt, 2011). Customer dissatisfaction with voucher expiration and other discount terms prompted ongoing tension as well as class-action litigation (i.e. Re Groupon Marketing and Sales Practices Litigation (2011)). In our view, the savviest analysts question the underlying value proposition for firms, asking whether large voucher discounts can genuinely be profitable, particularly given the large fees paid to voucher services.

Our model offers an important refinement of the prevailing understanding of the decline of voucher services. Specifically, our model indicates that a discount voucher service is more likely to be profitable for affiliated firms, all else equal, if customers using that service have valuations substantially different from (and in particular, below) those of other customers. Notice the difficulty as the voucher service grows in popularity: As more consumers use vouchers, voucher-users necessarily come to resemble average consumers. Consequently, as voucher services grow,
voucher-users become less likely to be consumers with systematically low valuations. Thus, as voucher services grow, they must rely more on advertising benefits rather than on price discrimination benefits. In a 2011 draft of this paper, we remarked that, in light of these factors, "Current voucher services' profits and recent growth may therefore not be good predictors of those services' future values." The subsequent decline of discount voucher services is consistent with our concern.

Meanwhile, we are struck by the large fees that leading discount voucher services charge to participating firms. Groupon's standard offer to firms is to charge $50 \%$ of voucher price: If a restaurant offers a $\$ 20$ voucher for $\$ 40$ of food, Groupon retains $\$ 10$. As our results in Section 3.1 indicate, such large fees may impede firms' usage of discount vouchers. Competition among discount voucher services has driven these fees down somewhat. Indeed, small voucher services may charge fees as low as $10 \%$. Yet small voucher services cannot provide substantial advertising benefits, potentially making their lower prices a false economy.

Ultimately, we wonder whether a single voucher service can satisfy diverse firm objectives. Some firms seek price discrimination to offer differing prices to consumers with varying willingness to pay. To implement price discrimination, these firms want to reach low-value consumers who do not overlap with the firm's existing customers (who the firm of course wants to continue to serve at full price). Meanwhile, other firms seek advertising services to help make their offerings broadly known. Such firms want to reach consumers similar to their existing customersprobably favoring high-value customers over low. Moreover, the offers to these groups may well differ: Facilitating price discrimination, a firm should accept multiple repeat visits from a low-value consumer, so long as the consumer's net payment on each visit exceeds the marginal cost. On the other hand, the advertising vision calls for a single discount followed by multiple full-price visits-potentially allowing a discount even below marginal cost (given the prospect of subsequent recoupment), but making it important to limit vouchers to one-per-customer. We sense that many voucher services have conflated these approaches, to the detriment of both. Long-term success of discount vouchers will require increased clarity on service function, targeting, and pricing.

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## Appendix

## Proof of Proposition 1

Equation (3) reveals $\gamma$ affects the magnitude of $\Delta \pi\left(p^{*}\right)$ the change in profits, but not the sign. Thus, if $\Delta \pi\left(p^{*}\right)>0$, then increasing $\gamma$ strictly increases the firm's profits.

## Proof of Proposition 2

To show the first part of the proposition, we suppose that $r^{G}=r^{F}, F(v)=G(v)$ for all $v$, and $\min \{\alpha, \beta\}<1$. We first note that it cannot be profitable for the firm to offer vouchers if $\beta<1$ and if $\alpha=1$, as that would reduce firm revenue without any change in the set of consumers making purchases. Thus, we suppose that $\alpha<1$ and $\beta \leq 1$. Then, then the firm is effectively offering price $\alpha p^{*}$ to fraction $\lambda$ of the $G$-population consumers and deriving revenue $\alpha \beta p^{*}$ from the $G$-population consumers who make purchases. If this is profitable for the firm (relative to not offering vouchers) then, by Proposition 1, it must be profitable for the firm to offer price $\alpha p^{*}$ to all $G$-population consumers, deriving revenue $\alpha \beta p^{*}$ from each of their purchases. But then, if $r^{G}=r^{F}$ and $F(v)=G(v)$ for all $v$, then the $G$-population consumers are identical to the $F$-population consumers. It must therefore be also profitable for the firm to to offer price $\alpha p^{*}$ to all $F$ population consumers, deriving revenue $\alpha \beta p^{*} \leq \alpha p^{*}$ from each of their purchases. This implies that the firm can increase its profits outright, pre-voucher, by offering price $\alpha p^{*}$ to all consumers-but this contradicts the optimality of $p^{*}$.

Now, to show the second part of the proposition, we suppose that

- if $r^{G}=r^{F}$ and $F(v)<G(v)$ for all $v \in V$,
- if $r^{G}>r^{F}$ and $F(v)=G(v)$ for all $v \in V$, or
- if $r^{G}>r^{F}$ and $F(v)<G(v)$ for all $v \in V$.

In this case, offering a voucher to the $G$-population corresponds to offering price $\alpha p^{*}$ to fraction $\lambda$ of the $G$-population consumers. As before, if this is profitable for the firm (relative to not offering vouchers) then, by Proposition 1, it must be profitable for the firm to offer price $\alpha p^{*}$ to all $G$-population consumers. The optimal $\alpha$ for the firm, which we denote by $\alpha^{*}$, solves

$$
\begin{equation*}
\left(1+\delta^{f} r^{G}\right)\left(\left(1-G\left(v\left(\alpha^{*} p^{*} ; r^{G}\right)\right)\right)-\frac{1+\delta^{i} r^{G}}{r^{G}+\delta^{i} r^{G}}\left(\alpha^{*} \beta p^{*}-c\right) g\left(v^{*}\left(\alpha^{*} p^{*} ; r^{G}\right)\right)\right)=0 \tag{8}
\end{equation*}
$$

By our assumptions on $r^{G}$ and $G$ and the fact that $p^{*}$ is set as the optimal price facing both populations (in (1)), we see that the optimal $\alpha^{*}$ in (8) in the case that $\beta=1$, which we denote by $\underline{\underline{\alpha}}$, is strictly less than 1 . As (8) is continuous, this implies that there is some $\underline{\beta}<1$ and $\underline{\alpha}$ with $\underline{\underline{\alpha}} \leq \underline{\alpha}<1$ such that offering a discount voucher with discounted price of $\alpha p^{*} \geq \underline{\alpha} p^{*}$ is profitable so long as $\beta \geq \underline{\beta}$.

## Proof of Proposition 3

If (6) holds, then

$$
0<\gamma \lambda(1-\kappa) \begin{cases}\left(\left(\alpha \beta p^{*}-c\right)+\delta^{f} r^{G}\left(p^{*}-c\right)\right)\left(1-G\left(\tilde{v}\left(r^{G}\right)\right)\right) & \alpha>r^{G} \\ \left(\alpha \beta p^{*}-c\right)\left(1-G\left(\hat{v}\left(r^{G}\right)\right)\right)+\delta^{f} r^{G}\left(p^{*}-c\right)\left(1-G\left(v^{*}\left(r^{G}\right)\right)\right) & \alpha<r^{G}\end{cases}
$$

It follows that there is some $\tilde{\kappa}>0$ such that the second term in (5) dominates the first at $\kappa=\tilde{\kappa}$. As (5) is linear in $\kappa$, the result then follows, with $\bar{\kappa}=\tilde{\kappa}$.


[^0]:    ${ }^{1}$ The proliferation of voucher discount services garnered substantial press: a multitude of newspaper articles and blog posts, and even a feature in The New Yorker (Surowiecki, 2010).

[^1]:    ${ }^{2}$ In a case study, Dholakia and Tsabar (2011) track a startup's Groupon experience in detail.
    ${ }^{3}$ Byers et al. (2012a) find that a merchant's rating on Yelp falls precipitously after that merchant offers a discount voucher. Byers et al. (2012b) suggest that this effect may be due to voucher-users' abnormally high likelihood of posting accurate (rather than biased or completely fake) reviews.
    ${ }^{4}$ Arabshahi (2011) considers vouchers from the perspective of the voucher service, whereas we operate from the perspective of participating firms. Gupta et al. (2012) and Norton et al. (2012) present general case studies of the voucher discounting business model.
    ${ }^{5}$ Firms know the distribution of consumer valuations and the (common) probability of fit.

[^2]:    ${ }^{6}$ While our treatment of advertising includes consumers not being aware of the firm's existence, conditional upon learning of the firm, consumers in our model receive more information than the firm does about their valuations. Additionally, our work differs from the classical literature on the advertisement of experience goods, as advertising in our setting serves the purpose of awareness, rather than signaling.
    ${ }^{7}$ In other models, heterogeneity in consumer search costs (e.g., Salop and Stiglitz (1977)) or reservation values (e.g., Sobel (1984)) motivate sales.
    ${ }^{8}$ We typically think of $\delta^{f}>\delta^{i}$, with firms more patient than consumers, although this is not required for our results.
    ${ }^{9}$ Modeling multiple consumer populations is a standard approach in the literature on personalized pricing. (See, e.g., Huang (2013); Deb and Said (2013).) Equivalently, we could consider a large consumer population with probability distributions over relevant characteristics. Our approach facilitates describing how "regular consumers" must differ from the consumers targeted by voucher offers in order for vouchers to be profitable for a firm.

[^3]:    10 In principle, voucher services are available to everyone. But in practice, some consumers cannot reasonably use vouchers or face significant impediments to doing so. Consider consumers without computers, consumers who disfavor electronic commerce, consumers who decline to buy prepaid vouchers, and consumers who find vouchers "tacky" or otherwise undesirable.
    11 Termes (2011) offers some supporting evidence for this fact.

[^4]:    ${ }^{12}$ For now, we assume the firm did not consider the possibility of a voucher when setting its price. In Section 4.1, we consider the possibility of price re-optimization.
    ${ }^{13}$ Introducing $\gamma$ lets us consider whether firms would want to limit vouchers to only a portion of the $G$-population. In Proposition 1 , we show that firms in fact do not want to do so. If $\gamma<1$, we assume that each consumer in the $G$-population has equal probability of being offered a voucher.
    ${ }^{14}$ If vouchers have word-of-mouth benefits, wherein voucher-users tell friends who then purchase from the firm, the first reference to $r^{G}$ in each component of (3) would be replaced by $r^{G}$ plus some factor $\omega$ corresponding to the number of friends referred by each voucher user. If voucher-users spend additional amounts $\eta$ beyond the voucher face value, then $p^{*}-c$ in each term should be increased by the additional profit from that additional spending. Each of these features would increase the relative profitability of discount vouchers.
    ${ }^{15}$ By contrast, $\lambda$ affects $p^{*}$ and can therefore affect the sign of $\Delta \pi\left(p^{*}\right)$.

[^5]:    ${ }^{16}$ There is no closed-form condition for this property in terms of the primitives of our model. Hence, in order to directly relate voucher profitability to model primitives (without imposing specific functional forms for $F$ and $G$ ), we must impose stronger conditions such as those used in the statement of Proposition 2.
    ${ }^{17}$ If $r^{G}=r^{F}$ then $\delta^{f}$ does not affect $p^{*}$, so there is no indirect effect and patience unambiguously increases the profitability of vouchers.

[^6]:    ${ }^{18}$ This approach facilitates comparison with the results of Section 3.1 because it implies (as we discuss below) that the firm's optimal price $p^{*}$ is unchanged. This approach is also conservative: If we allowed a consumer's probability of knowing about a firm to be correlated with the consumer's valuation for that firm, it would be natural to assume a positive correlation (consumers having higher valuations for the firm's product are more likely to be aware of it), which would tilt results against voucher services. (In light of our predominantly negative results on the benefits of voucher services, it is conservative to make assumptions that favor them.)

[^7]:    19 The only exception is if firms are much more impatient than consumers, with $\delta^{f}<\beta \delta^{i}$ and $\beta$ not too small, so that $\alpha \beta>\delta^{f} r{ }^{G}$ even though $\alpha<\delta^{i} r^{G}$.

