Pricing and Efficiency in the Market for IP Addresses

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Abstract

We consider market rules for the transfer of IP addresses, numeric identifiers required by all computers connected to the Internet. Excessive fragmentation of IP address blocks causes growth in the Internet's routing table, which is socially costly, so an IP address market should discourage subdividing IP address blocks more than necessary. Yet IP address transfer rules also need to facilitate purchase by the networks that need the addresses most, from the networks who value them least. We propose a market rule that avoids excessive fragmentation while almost achieving social efficiency, and we argue that implementation of this rule is feasible despite the limited powers of central authorities. We also offer a framework for the price trajectory of IP addresses. In a world without uncertainty, the unit price of IP addresses is constant until all addresses are in use and begins decreases at that time. With uncertainty, the price before that time is a martingale, and the price trajectory afterwards is a supermartingale. Finally, we explore the role of rental markets in sharing information about address value and assuring allocative efficiency.

1 Introduction

Every device connected to the Internet—from PCs to tablets, printers to cash registers—needs an IP address. The current addressing standard, IPv4, uses addresses with 32 binary digits, allowing approximately 4 billion IP addresses. The world's centralized supply of unused IP addresses reached exhaustion in February 2011, and networks in most countries will soon find they cannot easily obtain additional IPv4 addresses. While addresses may now be bought and sold, the institutions and rules of these transfers are not yet well-developed. Nor have economic models examined the unusual characteristics of this market. In this paper, we seek to speak to the latter gap.

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Low-cost electronic communications have sparked the transformation of many markets—both in creating the need to accommodate large communities of market participants, and in providing tools to make centralized markets mechanisms logistically viable. For example, keyword auctions let tens of thousands of buyers bid for millions of distinct objects—a context ill-suited to the negotiations traditionally used to sell advertising. (Edelman et al. (2007)) At the same time, the Internet facilitates market mechanisms that would have been too cumbersome without modern information technology. For example, the spectrum auctions considered in McMillan et al. (1998) require quick transmission of other bidders' activities. In this paper, we consider the market for IPv4 addresses—another example of the Internet expanding feasible market structure, and also a market that is important for the continued growth of the Internet. Meanwhile, in the spirit of papers which let the unusual requirements of a particular context guide the design of the corresponding allocation rules and institutions (e.g. school choice, Abdulkadiroglu et al. (2005), and kidney allocations, Ashlagi et al. (2011)), we consider novel rules in service of special concerns in IP markets.

Today the market for IPv4 is quite small—less than 100 transactions to date, covering a fraction of a percent of the Internet's address resources. (Mueller et al. (2012)) That said, there is reason to expect IPv4 transfers to become large and important both in dollar volume and in the critical role that IPv4 transfers will play in enabling continued growth of the Internet.

In this paper, we use economic theory to examine the special structure of the market for IPv4 addresses, including the relevant externalities, applicable restrictions, and the conditions that could support a near-efficient outcome. Our results are intended to assist address authorities in shaping transfer rules, as well as to guide the networks that will need to make trading decisions in this new market. We also seek to provide a basis for study by the economists and computer scientists likely to further examine this market.

We proceed as follows: In Section 2, we present the technical and institutional details that inform understanding of this market. In Section 3, we consider a one-period model of the market for IPv4 addresses and a mechanism that yields a (near) efficient outcome (notwithstanding the negative externalities we identify in Section 2.4). In Section 4, we offer an equilibrium framework for modeling the price trajectory of IP addresses over time; we examine the significance of private information in setting prices; and we consider the role of rentals in achieving allocative efficiency.

2 The technologies and institutions of IP addressing

2.1 The institutions of IP addressing

Evaluating feasible rules for IPv4 markets requires understanding responsible institutions. We summarize the most important facts here; Edelman (2009) offers more details.

Like most of the Internet's infrastructure, the IP addressing system is largely private. IP communication systems were initially developed pursuant to US government contract, but private parties have always administered address assignments. Under current allocation procedures, the Internet Assigned Numbers Authority (IANA), a California non-profit, assigns addresses to five regional internet registries (RIRs), which in turn assign addresses to ISPs and large networks. (In some regions, RIRs first assign addresses to country-specific internet registries, which in turn assign addresses to ISPs and networks.) Large networks tend to get their addresses directly from RIRs, while smaller networks and residential end-users get addresses from ISPs.

Because RIRs are private, they have little ability to penalize networks that flout their rules. Indeed, networks seem to value the limited powers of their RIRs, treating this as a philosophical matter (analogous to "limited government"). That said, RIRs do have some important powers. Notably, most ISPs check an RIRs records before accepting a new customer purporting to use a given block of IP addresses. Through "WHOIS" listings, RIRs report the network authorized to use a given block of IP addresses. Historically, WHOIS has been sufficiently desirable—and RIR rules sufficiently unobjectionable and well-regarded—that networks have been willing to comply with RIR rules in order to obtain official address assignments, WHOIS listings, and associated benefits.

To date, RIRs have examined each network's request to confirm the need for the requested addresses; addresses are issued only to networks that can justify their requests, e.g. through business plans, equipment purchases, and customer lists. Meanwhile, RIRs charge low fees. For example, the North American RIR, the American Registry of Internet Numbers (ARIN), charges just $18,000 per year to its largest ISPs.

With modest fees and rapid Internet growth, IPv4 resources are in short supply. IANA announced in February 2011 that it had exhausted its reserves. APNIC, the RIR for the Asia/Pacific region, finished its supply in April 2011; RIPE NCC (Europe) ran out in September 2012; and ARIN (North America) is expected to run out by fall 2013.

2.2 Technical responses to IPv4 scarcity

Facing limited availability of IPv4 addresses, growing or newly-created networks have several options:

- Networks can move devices to an alternative addressing system called *IPv6*, which offers 128 binary bits of numbering capacity and therefore is in abundant supply. However, a device with only an IPv6 address cannot directly communicate with IPv4-only devices; for example, it cannot directly access a web site that has only an IPv4 address. Translation systems are not yet widespread, and some protocols may be difficult to translate between IPv4 and IPv6, making IPv6 a more appealing solution when others have adopted it also. Furthermore, until IPv6 is in widespread use, IPv6 communications will take longer and slower routes (limited to passing among IPv6-enabled routers). In addition, some have found that IPv6 systems less reliable than IPv4 (typically for lack of around-the-clock monitoring and troubleshooting).

- Alternatively, networks can seek to *make do with fewer* IPv4 addresses, typically by installing network address translation (NAT) devices which allow multiple devices to share a single public-facing IP address. By rewriting packet headers, NAT maintains the illusion that all local devices have a single public address. NAT devices have become common in the home gateways that many homes install in order to connect multiple computers to a single DSL or cablemodem connection. While most protocols function as expected through NAT, others do not or require significant customization, and NAT impedes certain kinds of innovation. (Blumenthal and Clark (2001)) Furthermore, large-scale "carrier-grade" NAT would raise weightier questions of reliability and scalability, as well as reducing the likelihood of transition away from NAT in the coming years.

- Finally, networks can *buy* IPv4 addresses from others. Some networks may have more than they need—whether due to overly generous initial allocations, reduced requirements, or migration towards address-conserving alternatives (such as IPv6 or NAT). Meanwhile, others need more than they have—for example, new or growing networks.

In the long run, it is widely expected that networks will move to IPv6. But in the short run, v6 transition has been slow. For one, network effects encourage use of IPv4: with most users and sites on IPv4 only, it is natural to want IPv4 connectivity and unusual to want v6. Limited and untested translation systems—untested in part due to lack of customer demand—have further hindered transition. Meanwhile, NAT has not been deployed on the required scale and, even in the best case, adds complexity to network structure. For these reasons, it is currently anticipated that many new and growing networks will, in the short run, turn to IPv4 purchases to meet their IPv4 needs—prompting questions of the design of markets and institutions to facilitate such transfers.

2.3 The prospect of paid transfers of IPv4 addresses

Historically, paid transfers of IP addresses were impermissible. After all, with sufficient addresses available directly from RIRs, there was no need for sales between networks, and any buyer was either a scofflaw (e.g. a spammer needing limitless new addresses to send unsolicited email—a group networks and RIRs did not seek to assist) or a fool (who would be better served by a strong norm of obtaining addresses directly from RIRs). But as scarcity loomed, RIRs revised their policies to allow sales. For example, the ARIN transfer policy is codified in Number Resource Policy Manual (NRPM) at section 8.3.

2.4 A negative externality from IPv4 sales

While trade in IPv4 addresses promises various benefits, including those sketched in preceding sections, transfers also prompt concerns. Most common is the fear that certain address transfers might threaten the Internet's routing system—the infrastructure that transfers data from one network to another. The Internet's routing architecture requires that each "default-free zone" (DFZ) router on the Internet keep a record of each block of addresses used anywhere else on the Internet. These records are stored in each router's routing table—high-speed memory limited in size and limited further by the need to process and search the routing table exceptionally quickly. Importantly, a small block of addresses requires just as much routing table capacity as a large block. In particular, any block of addresses, of any size, requires one routing table row to indicate how to send traffic destined for that block of addresses. If large networks begin to acquire many small address blocks, rather than a few large blocks, the routing table could grow sharply. For example, a network might elect to buy 16 blocks of 256 addresses, rather than one block of 4,096, if the former is slightly cheaper. In that case, growth would be an order of magnitude more rapid.

At best, growth in the routing table would require that networks upgrade their routers more frequently. But particularly rapid growth could exceed router manufacturers' ability to improve their offerings—destabilizing the Internet's routing system.

No single entity—neither public nor private—exerts meaningful control over the routing system. As a result, no one can easily impose rules on what routes may be added to the routing table, nor is there anyone to collect a fee for each addition. In North America, ARIN initially established rules requiring each buyer to satisfy the entirety of its short-run need in a single transfer—disallowing multiple small transfers, and thereby avoiding unnecessary growth of the routing table. See ARIN NRPM rule 8.3, initially allowing transfers only to buyers which "can demonstrate the need for such resources, as a single aggregate, *in the exact amount* which they can justify under current ARIN policies" (emphasis added). However, other RIRs took a different approach. For example, APNIC's transfer rules included no restrictions to prevent many small purchases. In a 2011 change, ARIN proposition 144 removed the "exact amount" requirement, allowing buyers to satisfy their needs via a series of small purchases.

In Section 3.2, we examine the welfare and efficiency properties of a rule that generalizes ARIN's "exact amount" requirement.

3 One period model and market mechanism

In most asset markets, it is typical for large buyers and large sellers to split their purchases and sales into small pieces to be bought or sold separately. An unusual feature of market for IPv4 addresses is that such splits are socially costly for the reasons presented in Section 2.4. We therefore seek to devise market rules to discourage unnecessary splits, and in this section we develop a formal model to evaluate the performance of such rules.[1]

3.1 Notation and definitions

Consider a set of networks, each with an endowment of IPv4 addresses. We will use the size of the smallest tradable block of IPv4 addresses as the unit of measurement, so when we say that network k bought x_k addresses, we mean that k bought x_k times the smallest block. $x_k < 0$ means that k was a seller.

Denote by $f_k(x)$ the value that k derives from increasing (or the cost k incurs in decreasing) its IPv4 address holdings by x. Normalize $f(0) = 0$. We assume that $f()$ is increasing. We also assume decreasing differences—that the marginal benefit of an address decreases in the number of addresses the network holds (the discrete analogue of a negative second derivative). Formally, we require $f(x) - f(x-1) \leq f(x-1) - f(x-2)$ for all x. This condition is needed to ensure the existence of the competitive equilibrium price.

Definition 1 *An* outcome *is a vector specifying how much each buyer purchases and how much each seller sells.*

Definition 2 *An outcome is* feasible *if* $\sum_k x_k = 0$ *and* $x_k \in I$ *for every* k.

Definition 3 *An outcome is* efficient *if* $\sum_k f(x_k)$ *is maximized subject to the feasibility constraints.*[2]

Due to the negative externality presented in Section 2.4, we will consider restrictions on trading to prevent excessive fragmentation of IPv4 blocks without inhibiting efficiency. The following definitions will assist in developing and analyzing such restrictions:

Definition 4 *Under the* spartan rule*, the two agents engaging in a bilateral trade must designate either the buyer or seller as "extinguished." The spartan rule prohibits an extinguished agent from trades with other extinguished agents. An agent can be extinguished at most once.*

Notice the relationship between our concept of an "extinguished" agent and the "exact amount" requirement from Section 2.4: Whereas "exact amount" required that every buyer's first trade be his last, our spartan rule allows either buyer or seller to be "extinguished" by a transaction, such that either the buyer or the seller ceases to trade further after that transaction. In Section 3.2 we examine the welfare and efficiency implications of this modified rule.

Because we offer a one-period model, an agent is extinguished forever. However, in practice IP allocations typically operate on a rolling basis, i.e. providing

[1] In a paper entitled "Minimizing Setup and Beam-On Times in Radiation Therapy", Bansal et al. (2006) considers a mathematically similar model, albeit in an entirely unrelated substantive context.

[2] This definition of efficiency does not consider the negative externality presented in Section 2.4.

a network with the addresses it can justify for a six month period. As a practical matter, it would be logical for extinguished status to last for that same duration.

A sequence of trades satisfies the spartan rule if and only if each agent is extinguished at most one time, and if each agent has at most one trade with an agent extinguished by someone other than himself. Consequently, if a sequence of trades satisfies the spartan rule, then the same trades in a different sequence also must satisfy the spartan rule. Thus, we can define and evaluate spartan allocations without considering the order of trades.

An allocation is an outcome and the set of bilateral trades that lead to that outcome. More formally:

Definition 5 *An* allocation *specifies which seller(s) each buyer was matched with, the size of each transaction, the price of each transaction, and which party was designated as extinguished.*

An allocation is spartan if each agent is extinguished at most one time. In that case, regardless of the order of trades, the spartan rule is satisfied.

In preparation for addressing the externality presented in Section 2.4, we offer a notion of subdivision of a seller's resources:

Definition 6 *A* cut *is a subdivision of a resource pursuant to a sale.*

Remark 7 *If a seller sells a positive quantity of resources to n different buyers, the number of cuts required is n (if the seller keeps some of the resource for itself) or $n-1$ (if the seller sells all of the resource).*

3.2 Welfare and efficiency under spartan allocations

Let N denote the number of buyers and K the number of sellers.

Lemma 8 *A spartan allocation involving N buyers and K sellers never entails more than $N+K$ cuts.*

Proof. With N buyers and K sellers, the maximum total number of transactions is $N+K$. (Either a buyer or a seller must be extinguished in each transaction, and each entity is extinguished at most once, so the number of trades cannot be more than the number of participating entities, $N+K$.) Suppose seller k was matched to n_k buyers. Since the total number of transactions is no more than $N+K$, it must be the case that $\sum_{k=1}^{K} n_k \leq N+K$. ■

Proposition 9 *For any feasible outcome, there exists a spartan allocation that induces that outcome.*

Corollary 10 *Suppose an allocation includes N buyers and K sellers. There exists a spartan allocation with no more than $N+K-1$ cuts that induces an efficient outcome.*

Proof. Consider an outcome involving N buyers and K sellers. Denote by V_i^0 the quantity obtained in that outcome by buyer i, and denote by W_j^0 the quantity sold by seller j. Consider a sequential mechanism where there is exactly one transaction in each period. Denote by V_i^t and W_j^t the number of units that buyer and seller respectively will acquire (or provide) in transactions that take place after period $t-1$. (If $V_i^t = 0$, than by period t buyer i already made all his trades, hence i is extinguished.) Consider the following sequence of trades satisfying the spartan rule: In period t, find the buyer with the smallest remaining demand. (Buyer k has the smallest remaining demand if $V_k^t \leq V_j^t$ for any j.) Similarly, find the seller with the smallest remaining supply at period t. ($W_m^t \leq W_j^t$ for any j.) In period t, match this buyer k with seller m. If $V_k^t > W_m^t$ (buyer k wants more than seller m can offer), they transact amount W_m^t—exhausting m's supply, leaving $W_m^{t+1} = 0$, while k still seeks additional resources $V_k^{t+1} = V_k^t - W_m^t$. Alternatively, if $V_k^t \leq W_m^t$ (seller m wants to sell more than buyer k requires) then in period t the transacted amount is V_k^t. In other words, in each period the smallest buyer and the smallest seller transact with each other the largest possible amount, thereby extinguishing either the buyer or the seller in each transaction. The sequence of trades constructed through this process can implement any outcome (including the efficient outcome) with $N + K - 1$ cuts. ■

Proposition 9 shows that even with the constraint provided by the spartan rule, any allocation remains feasible. Since we allow transfers between players, an inefficient allocation cannot be coalition-proof because the coalition containing all the players can move to an efficient allocation and arrange for transfers to make every agent better off, according to the standard argument. This yields the following corollary:

Corollary 11 *Any coalition-proof spartan allocation is efficient, and there exists a competitive equilibrium price that supports it.*

3.3 Properties of minimal allocations

The preceding section characterizes the maximum number of cuts required for spartan allocations and efficient outcomes. In this section, we examine the *minimal* number of cuts in order to minimize the externality flagged in Section 2.4.

Definition 12 *A* minimal allocation *is an efficient allocation with weakly fewer cuts than any other efficient allocation.*

Proposition 13 *There exists a spartan allocation that is also minimal.*

Proof. Suppose every efficient spartan allocation results in more cuts than an efficient allocation with the smallest number of cuts (a minimal allocation). First, note that there are many minimal allocations because without the spartan rule, the identity of the party extinguished in each trade can be assigned arbitrarily. An allocation can be viewed as a graph with connections between agents who trade and with arrows pointing away the agent extinguished in each trade. Without loss of generality we can consider an allocation that has only one connected component. Indeed, if there are multiple connected components, the problem becomes separable because there is an outcome corresponding to each component and for each component it must be impossible to reduce the number of cuts without changing the outcome. Thus, without loss of generality we will assume that minimal allocation has only one connected component.

Suppose there does not exist a spartan allocation that leads to as few cuts as a minimal allocation. Then for any minimal allocation, there must exist an agent who is extinguished more than one time. (Otherwise there would exist a spartan allocation that coincides with a minimal allocation.)

Consider a minimal allocation with the smallest possible number of unextinguished agents. An unextinguished agent must be directly connected to an agent who is extinguished only one time. (If a partner of an unextinguished agent were extinguished more than once, then we could reduce the number of unextinguished agents, a contradiction.)

The number of extinguishings equals the number of trades, so the total number of extinguishings is at least $N+K+1$ where N and K are the numbers of buyers and sellers respectively. One seller participates in every trade, and the number of cuts is the total number of trades minus the number of sellers, thus we showed that the number of cuts in the minimal allocation is at least $N+K+1$. But this contradicts Lemma 8 which showed that there are at most $N+K$ cuts in a spartan allocation. Hence, there must exist a spartan allocation that is minimal. ■

Corollary 14 *The number of cuts in the minimal allocation equals the number of buyers and sellers minus the number of (disjoint) components in the graph representing the minimal allocation.*

Proof. It follows from Corollary 10 that the k-th component has at most $N_k + M_k - 1$ cuts. Summing over all components, we obtain the upper bound on the number of cuts. Now let us show that a component of a graph induced by minimal allocation cannot have less than $N+K-1$ cuts. From Proposition 13, it follows that without loss of generality we can consider a minimal allocation that is spartan and hence all agents are extinguished no more than once. Suppose a component has fewer than $N+K-1$ cuts. In that case the total number of links is at most $N+K-2$ and hence at least two agents remain unextinguished. The trading partners of the unextinguished agents are extinguished (by transactions with unextinguished agents). Denote by Ω_{1h} and Ω_{2h} the sets of agents who are h links away from unextinguished agents 1 and 2. (Ω_{10} is agent 1 himself, and Ω_{11} is the set of agents who are connected to agent 1 and hence extinguished by that trade.) Note that agents in a set Ω_{1k} are extinguished by agents in a set Ω_{1k-1}. Consequently, any agent connected via a chain with agent 1 is extinguished by agents closer to agent 1, and the same is true for those surrounding other unextinguished agents. Thus, the clusters around unextinguished agents can never be connected to each other, which contradicts the assumption that they belong to the same component of the graph. ■

The preceding results establish the favorable characteristics of the spartan rule: it achieves efficiency and achieves the fewest possible number of cuts. At the same time, the spartan rule generalizes the ARIN rule presented in Section 2.4. Relative to the mechanism ARIN adopted, the spartan rule is more flexible in that it allows large buyers to more readily find the addresses they seek, yet it does so without inviting undue growth in the routing table.

4 Price trajectory

The previous section considers a static world: from the outset, all networks know their values for IPv4 addresses, and these values do not change over time. The preceding section ignored dynamic aspects of the market for IPv4 addresses. But in fact IP addresses are long-lived assets, and networks' needs change over time. In this section, we consider the dynamics of the IPv4 market, characterizing the price trajectory of IPv4 addresses.

In the previous section, we showed that even with integer constraints and even with the trading constraint of the spartan rule, the same efficient allocation obtains as in an unrestricted commodity market. Hence, for technical convenience this section assumes that IPv4 addresses are homogeneous and perfectly divisible.

Our examination of pricing builds on an understanding of long-term prices: Although IPv4 addresses are currently valuable, it is expected that scarcity will eventually be relieved as networks migrate to IPv6. In particular, once much of the Internet supports v6, there will be less need for IPv4 addresses, though supply with remain exogenous and fixed. As a result, it is reasonable to assume that at some period, T, the value of IPv4 address will become zero. But what happens to prices en route to that point?

To explicitly model the pricing process, we treat IPv4 addresses as a production input. Let $f_{it}(x_{it})$ denote the value that network i derives from having x_{it} more IPv4 addresses in period t than the network was endowed with before the first period. (If x_{it} is negative, the network is a seller.)

4.1 Price Trajectory in the Complete Information Case

We begin by considering a world without uncertainty or private information. (In section 4.2 we consider a world where information about demand for IPv4 arrives over time.)

We assume that the marginal value derived from an additional IP address is non-negative and diminishing, that is $f'_{it}(x) \geq 0$, $f''_{it}(x) \leq 0$, for all $x \in R$, $i \in \{1, ..., N\}$ and $t \in \{1, ..., T\}$. The supply of IPv4 addresses is exogenously fixed, that is $\sum_{i=1}^{N} x_{it} = 0$ for any $t \in \{1, ..., T\}$. At the start of each period, networks can buy and sell IPv4 addresses, and we will show that there is a unique competitive equilibrium price p_t in each period. The payoff of network i is the sum of period payoffs $\sum_{t=1}^{T-1} f(x_{it})$ plus the proceeds from buying and selling IPv4 addresses.[3]

The competitive equilibrium price trajectory of IPv4 is a vector of prices $p_1, ...p_T$ such that the market clears in each period if all networks trade in each period in order to maximize the sum of payoffs across periods. More formally, assuming that each network solves the maximization problem $\max[\sum_{t=1}^{T} f_{it}(x_{it}) - p_1 x_{i1} - \sum_{t=2}^{T}(p_t(x_{i,t} - x_{it-1}))]$, we will say that the price vector clears the market if there exists a solution $x^*_{it}(p_1...p_T)$ to the individual maximization problem such that $\sum_{i=1}^{N} x^*_{it} = 0$ for every t.

Proposition 15 *In a world without uncertainty, there exists a unique competitive equilibrium (CE) price vector $p_1...p_T$ with the following properties:*

1. *The price is non-increasing over time. That is, for any t and τ, if $t < \tau$ then $p_t \geq p_\tau$.*
2. *Prices are constant when addresses are not scarce. In particular, if during period t there exists at least one network that does not utilize some of the addresses that it owns (i.e. because $f'_{it}(x_{it}) = 0$), then $p_t = p_{t+1}$.*

Proof. We begin by showing that a competitive equilibrium exists. Rearranging and using the assumption that $p_T = 0$, a network's maximization problem can be rewritten us $\max[\sum_{t=1}^{T-1} f(x_{it}) - x_{it}(p_{t+1} - p_t)]$. Thus, the maximization problem is separable, with the network maximizing $f(x_{it}) - x_{it}(p_{t+1} - p_t)$ in each period. In particular, if the competitive equilibrium price vector exists, then the optimal course of action in period t is independent of actions taken in other periods. We can interpret $g_t = p_{t+1} - p_t$ as the rental rate of IPv4 addresses in period t.

Note that in a competitive equilibrium, $g_t = 0$ if and only if there exists a vector of $x^*_{1t}...x^*_{Nt}$ such that $f'_{it}(x^*_{it}) = 0$ for all i and $\sum_{i=1}^{N} x^*_{it} \leq 0$. In this case, there is no scarcity of IPv4 in period t. In a world without scarcity, the existence of a competitive equilibrium is obvious.

Now consider the case of scarcity and $g_t > 0$. We showed above that in a deterministic competitive equilibrium, a network's profit maximization problem is reduced to $\max(f_{it}(x) - xg_t)$ in each period. Note that the assumptions for $f_{it}(.)$ guarantee that the profit maximizing quantity x exists and is unique for every value of $g > 0$. Thus we can consider a demand function $x_{it}(\dot{g})$ that assigns to each positive rental price the corresponding demand by network i in period t. Our assumptions on f_{it} guarantee that demand is continuous and non-increasing in rental price $\dot{g}$.

We now demonstrate that the price trajectory is non-increasing. Consider the equilibrium rental price. If there is no scarcity in period t, then the competitive equilibrium rental price is zero. For the case of positive rental price, we previously established that $x_{it}(g)$ is continuous and non-increasing. If there exists a unique competitive equilibrium price in period $t+1$, there exists unique

[3] By adding dollar payoffs from different periods, we implicitly assume that the interest rate is zero.

equilibrium rent in period t and hence there also exists a unique competitive equilibrium price in period t. By assumption there is no scarcity in period T and all subsequent periods, which means that the competitive equilibrium price in period T must be $p_T = 0$. We showed that if there is a unique competitive equilibrium price in period T, there must exist a unique competitive equilibrium price in the previous period, and recursing across previous periods implies that the price in period t equals the sum of rental rates in the remaining periods $p_t = \sum_{\tau=t}^{T-1} g_\tau$. Since rents are non-negative, the price is non-increasing in time.

Finally, consider periods in which there is no scarcity of IPv4 addresses. Because rent equals zero in those periods, the price of IPv4 addresses does not change during such periods. ■

We recognize that there is not currently a formal rental market for IPv4; such a market may or may not be desirable, and may or may not emerge in the future. Regardless, the rental market is a useful framework for thinking about pricing of IP addressees. For example, if such purchases and sales were fully permitted by applicable rules, the act of buying an IP address in period t and selling it in period $t + 1$ can be interpreted as renting the address for period t at a price g_t.

We pause to offer intuition on the preceding results. Our analysis began with the insight that in any period without scarcity, the rental price of IPv4 must be zero. In other words, as long as there are blocks of IPv4 addresses that are not in use, equilibrium behavior of market participants requires that the rental price of IPv4 must be zero. (Of course, a zero *rental* price does not mean the prevailing market price of IPv4 should be zero. Rather, the market price of an IPv4 address incorporates expectations about future rental prices of that address.) Importantly, the efficient market hypothesis implies that an asset with zero per-period rental income must yield a market rate of return. In a world with a risk-free interest rate of zero and with no uncertainty, the price of IPv4 addresses must remain constant as long as there are some unused IPv4 blocks.

In contrast, consider the first period when there are no unused IPv4 blocks available. In this period, true scarcity begins, and the implied rent of an IPv4 address becomes positive. Notably, the market price of an IPv4 address declines each period by an amount equal to the implied rent of that address during the prior period. Using the preceding analyses, Figure 1 presents plausible price profiles for the complete information case (without uncertainty).

We believe our analysis of IPv4 price profiles is likely to be counterintuitive to non-economists. Non-economists might be surprised to hear that prices are at their highest before scarcity actually sets in, and that prices only decrease as networks search for sources of the scarce resources at issue.

4.2 Price Trajectory in a World with Uncertainty

In a world with uncertainty, the price of IPv4 addresses may increase or decrease as new information arrives. For example, prices would likely be influenced by new information about growth in the number of Internet users or changes in the speed of implementation of IPv6.

We model the arrival of new information by assuming that in each period there is a public signal s_t that influences demand for IPv4 addresses. Networks receive the signal s_t in the beginning of period t, then trade addresses as desired. The signals are jointly distributed according to a distribution $\rho(s_1, s_2..., s_T)$ that is common knowledge. We assume that ρ is well-behaved in the sense that all conditional distributions exist and are integrable. We denote by $f_{it}(s_1...s_t, x_{it})$ the payoff of a network i in period t, which depends on the history of signals and on the number of IPv4 addresses that the network already holds. Other assumptions remain as in the prior section.

The definition of competitive equilibrium price needs to be modified to include uncertainty. We now define a competitive equilibrium price as a set of functions $p_t(s_1, s_2...s_t)$ that clear the market for every realization of signals in each period, provided all networks maximize their expected payoff. We use a similar argument as in the previous section to establish properties of competitive equilibrium.

Proposition 16 *In a world where publicly observable signals about the value of IPv4 arrive over time, there exists a unique competitive equilibrium (CE) price vector $p_t(s_1, s_2...s_t)$ for $t = 1...T$ with the following properties:*

1. *Expected prices are non-increasing over time. That is, $E[p_t] \geq E[p_{t+1}]$.*

2. *Expected prices are constant when addresses are not scarce. If during period t there exists at least one network that does not utilize some of the addresses that it owns (i.e. because $f'_{it}(x_{it}) = 0$)), then $E[p_t] = E[p_{t+1}]$. As new information arrives during a period when addresses are not scarce, the price trajectory is martingale.*

Proof. We follow the proof of Proposition 15 but generalize to a world with uncertainty.

Consider a network's optimization problem in period $T-1$. By assumption, migration to IPv6 eliminates scarcity of IPv4 beginning in period T, so in period $T-1$ there is no remaining uncertainty: Everyone knows the price in the next period will be zero. With no uncertainty, Proposition 15 applies from then forward, guaranteeing that a competitive equilibrium price, $p_{T-1}(s_1, s_2, ...s_{t-1})$, exists and is unique.

Now consider an agent deciding how many IPv4 addresses to acqure in period $T-2$. The agent already knows signals $s_1...s_{T-2}$ but does not know the realization of the signal in period $T-1$. Consequently, from the agent's point of view, p_{T-1} is a random variable: for any realization of s_{T-1}, the agent knows the value of p_{T-1}, and the agent also knows the joint distribution of all signals and hence knows the distribution of the signal in period $T-1$ conditional on previous signals. Thus, given signals $s_1, ...s_{t-2}$, the expected value of the competitive equilibrium price in period $T-1$ is well defined, and we denote it by $E[p_{T-1}|s_1...s_{T-2}]$.

Let us define $g_{T-2} = p_{T-2} - E[p_{T-1}|s_1...s_{T-2}]$, which we interpret as the (expected) rental price for a period. Then an agent in period $T-2$ considers the maximization problem $\max[f_{it-2}(s_1, s_2, ...s_{t-2}, x_{it-2}) - x_{iT-2}g_{T-2}]$. Thus period $T-2$ demand is a function of the rental price, and the problem matches the world of certainty, save only for dependence on the expected rental rate in period $T-2$ (rather than the actual rental rate in the case without uncertainty).

We have shown that if there exists a competitive equilibrium price in period t, then there must exist a competitive equilibrium price in the previous period $t-1$. By assumption, the competitive equilibrium price in period T is zero, so the competitive equilibrium price exists and is unique, and the expected decline in price in period t is equal to the rental price in period t. ■

4.3 The Benefits of a Rental Market and Constraints on IP Address Rentals

The preceding sections characterize the price trajectory in a world with public availability of all information influencing the price of IPv4 addresses. But in practice, some information will typically be private to individual participants. Depending on the information structure available to market participants, a rental market can assist in spreading information and putting IPv4 resources to use.

Finance, general equilibrium and industrial organization models often assume that prices correctly aggregate private information, thus leading to allocative efficiency. In many contexts, there are good reasons to expect that markets aggregate information and achieve allocative efficiency. Indeed, work dating to Adam Smith recognizes that markets aggregate information, and a series of papers have formalized the mechanisms of information aggregation. It is well known that in a private value setting, an English auction leads to allocative efficiency. In a common value setting, Ostrovsky (forthcoming) shows that, if trading is frictionless, prices correctly aggregate all private information (even if agents obtain multidimensional private signals). However, in environments with both common and private value components, markets may fail to aggregate information, a possibility first explored by Maskin (1992).

There are good reasons to suspect that IPv4 address transfers may not reach an efficient allocation. We can think of the value of IPv4 address to a network as a sum of the network's private value of using it during the present period plus the resale value in a future period. The value from renting an address is a private value component and the resale value in future periods is a common value component. Efficient information aggregation results do not apply in this context due to the presence of both common value and private value components, with participants having private information about both. In particular, each network will have private information about its private value for IPv4 addresses in the current period as about its future demand for IPv4. Some networks may also have greater insight into factors affecting the common value of IPv4 addresses. For example, thanks to their special roles in providing or deploying transition technologies, some networks may enjoy superior information about

IPv6 deployment and about IPv4 address sharing. More generally, networks have private signals containing information about both their individual private value for IPv4 in the present period as well as information about future demand for IPv4.

The following example illustrates why IPv4 transfers may not lead to an efficient outcome. Consider a network that has unused IPv4 addresses. If the network has unused addresses after scarcity occurs, declining market prices provide an incentive for the network to sell the addresses quickly to avoid the price drop that results from the positive real rental price. However, suppose the network also has a private signal about the common value. For example, the network might believe that future demand will be higher than currently anticipated by most other market participants. With this private information, the network seeks to hold IPv4 resources in anticipation of future price increases not expected by others. Note that the market will fail to aggregate this network's information because others cannot tell whether the network is accumulating IPv4 because it has high private value for using them in the current period or because it has private information that the price will increase in future periods. As a result, a network may elect to speculate in IP addresses by continuing to hold addresses that it does not use.

In principle, a rental market can help spread information and put IPv4 resources to their highest and best use. Returning to the problem posed in the preceding paragraph: If a rental market existed, the network could rent its unneeded addresses—collecting the positive per-period rental price and achieving allocative efficiency by putting scarce resources to use, yet continuing to enjoy the benefit of possible increase in the market price. In contrast, without a rental market, the network's private information is not passed to the market, and prices do not include the network's information. The essence of the problem is that address value includes both private value and common value components. The availability of a rental market would absorb the common value component, allowing market participants to trade on their private signals.

Some aspects of IP addresses seem to lend themselves to a rental market: For example, there is little prospect of latent or concealed damage to an IP address. To the extent that IP addresses can be damaged, via spamming or other harmful behaviors that reduce the reputation of an address block, most such reputation information is quickly and publicly available. Compare concealed damage to a home or automobile, markets where rentals have proven problematic for some participants. With address condition fully observed to market participants, providers and renters can write contracts to disallow damage and require payment in case of damage.

That said, other aspects of IP addresses make rentals difficult. For example, there is no strong central authority or other obvious process to "repossess" rented IP addresses at the conclusion of a rental. Indeed, if a renter continued to use the rented addresses and continued to announce their use via the Internet's routing system, the provider would have little ability to reclaim the addresses from the renter. Conflicting address announcements test the stability of the routing system, but in any event most networks would resolve conflicts in favor of whichever network provides more useful and better-known content on a given address block—a rule that tends to favor recent use over contract rights. A proposed system of cryptographic verification of routing rights, Resource Public Key Infrastructure (RPKI), has not yet been deployed by network operators and has prompted concerns about reliability, security, and overly-centralized authority. Finally, community norms may also stand in the way: many ARIN members expressed concern at the prospect of allowing IP addresses to be bought and sold, and rentals would surely raise more questions. In the short run, it seems rentals will probably remain difficult and limited.

Policy discussions at ARIN suggest a commitment to putting IPv4 resources to use, avoiding waste, and making transactions easy and safe for participating networks. It seems rentals could advance these objectives: For example, a vibrant rental market would let a network provide its unneeded addresses for use by others, even if the network anticipates a future increase in market prices (and therefore does not want to sell the addresses in toto). Furthermore, rentals could help prevent predictable changes in prices—discouraging efforts to try to "time the market" in buying or selling addresses. Policy changes could help facilitate these benefits. For example, a RIR could offer time-limited WHOIS updates, letting a network provide addresses for temporary use by others, with WHOIS guaranteed to revert after some predefined period.

The importance of rental markets extends beyond the market for IPv4 addresses. In general a durable good cannot be allocated efficiently if the only way to trade on information about future value is to hold the good in the current period; in that case, someone without a productive use for the good may nonetheless hold it in anticipation of price increases, to the exclusion of others who could immediately put it to productive use. Consequently, even if the market for the good is frictionless, a rental market is needed to achieve efficient allocation. In principle, these concerns are as true for, e.g., real estate as for IP addresses. In practice, real estate markets have already developed mechanisms to facilitate rentals—for example, well-established rules for evicting a renter whose rights have expired. The lack of similar institutions for IP address rentals is likely to stymie such rentals. Meanwhile, real estate rentals entail genuine and inevitable frictions (moving costs, possibility of concealed damage, etc.), which pose important limits on short-term rentals in that context. In contrast, for many networks it is straightforward to move from one IP address range to another. Thus the potential benefits of IP address rentals should be particularly large, if institutions develop to support such rentals.

4.4 The Current State of the IPv4 Market

Whatever the price trends and trajectories, networks' most immediate concern is typically the *level* of prices—an urgent question for networks seeking to plan future expansion. RIRs report the resources that have been transfered through their paid transfer systems, facilitating the tabulation and analysis in Mueller et al. (2012)—reporting a total of 83 transactions totaling 6 million IPv4 blocks through June 2012. That said, RIR records lack information about prices: RIRs have no operational reason to collect prices, and to date no RIR has required networks to disclose prices as a condition of transfer.

To date, price data is available only for a subset of transactions: sales from networks in bankruptcy proceedings. Consistent with standard practice of litigation records open to public review, bankruptcy filings reveal the amounts received for debtors' assets, including IPv4 addresses. In one widely-publicized 2011 transaction, Microsoft paid $7.5 million for approximately 666,000 IPv4 addresses—$11.25 per address. Other transactions from sellers in bankruptcy have brought prices that are broadly similar.

It might seem natural to use initial transactions to predict future prices. But we question whether sales from bankrupt estates are useful in predicting prices. By all indications, early sales reflect buyers and sellers in unusual conditions. For example, as of November 2012, networks could still obtain IPv4 addresses from ARIN to satisfy up to three months of documented need, at prices that are effectively zero. With addresses available from ARIN at minimal expense, why would a network pay a positive price in a paid transfer? Transfers let networks buy addresses to satisfy 12 to 24 months of need, but even so a network would pay a hefty premium relative to ARIN's near-zero prices—calling into question whether the buyers that make early purchases on the transfer market are representative of other buyers. Mueller et al. (2012) and others have suggested that a seller in bankruptcy can transfer greater rights in IPv4 resources than an ordinary seller in an ordinary arms-length transaction—a claim that ARIN disputes, but a rationale that might support a higher price for resources sold in bankruptcy. Meanwhile, early sellers also appear atypical: A bankrupt seller is unable to await the more vibrant market that might arise in the future—suggesting that bankrupt sellers may be behaving suboptimally. In short, we doubt that these early transactions fit a simple model of IPv4 price trajectory, and we question whether these early sales well predict the future price of IPv4 addresses.

5 Concluding remarks

The impending scarcity of IPv4 addresses calls for economic research to facilitate suitable institutions and market rules. In many markets, participants had the benefits of a period of years to design such systems, and often geographically-isolated markets served as parallel laboratories to invent and test alternative approaches. In contrast, IPv4 addresses have been issued at de minimis price and will continue to be issued that way until RIRs have fully exhausted their supply—delaying the development of market institutions. Then, IPv4 scarcity will arrive in one fell swoop, raising the stakes for any adjusting of rules and offering less opportunity to refine rules over time. Furthermore, because IPv4 resources are inherently portable, experiments in one region would tend to affect behavior elsewhere—limiting the potential for regional innovation. In these circumstances, economic theory can provide particularly valuable insights on market design.

Our proposed spartan rule builds on ARIN's initial restriction—seeking to address the same negative externality, but offering somewhat more flexibility in order to avoid unnecessary deviations from efficient allocations. We believe ARIN could adopt our rule within its existing transfer framework and with minimal administrative burden.

As IPv4 markets develop, there will also be an opportunity for empirical economic research. For example, will prices in fact be linear in block size, or will large blocks carry disproportionately higher (or lower) prices? Combining such observations with relevant economic theory could yield prompt diagnoses of market malfunctions and timely interventions to preserve efficiency while limiting negative externalities.

Figures

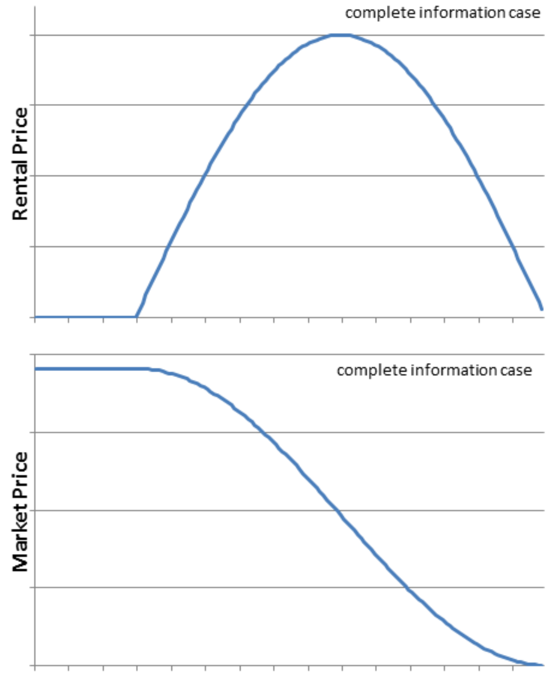

Figure 1: Price Profile of IPv4 Addresses

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