Price Coherence and Excessive Intermediation[*]

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Abstract

Suppose an intermediary provides a benefit to buyers when they purchase from sellers using the intermediary's technology. We develop a model to show that the intermediary would want to restrict sellers from charging buyers more for transactions it intermediates. With this restriction an intermediary can profitably raise demand for its services by eliminating any extra price buyers face for purchasing through the intermediary. We show that this leads to inflated retail prices, excessive adoption of the intermediaries' services, over-investment in benefits to buyers, and a reduction in consumer surplus and sometimes welfare. Competition among intermediaries intensifies these problems by increasing the magnitude of their effects and broadening the circumstances in which they arise. We discuss applications to payment card systems, travel reservation systems, rebate services, and various other intermediaries.

JEL classification: D40, L11

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1   Introduction

Sellers often choose to provide their goods and services via intermediaries which serve as distributors, brokers, payment processors, or other facilitators. Ideally, intermediaries have a genuine advantage— perhaps superior knowledge of local conditions, lower costs, better user interface, or a complementary

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benefit to better serve buyers' needs. That said, certain intermediaries' pricing policies can lower consumer surplus below the level that would result without those constraints. Moreover, intermediaries can thrive even when they offer little or no actual value due to a market failure caused by the structure of the relationship between buyer, seller, and intermediary.

Specifically, the purchase of a given good via an intermediary is often constrained to occur at the same price as a purchase of that same good directly from the seller or through a competing intermediary, which prevents the buyer from considering the cost of the intermediary's service. For example, a merchant may be obliged to charge the same price to customers who pay cash as those who pay by credit card; an airline, to charge the same price to a customer who books directly or via a travel agent. We call this constraint "price coherence," following Frankel (1998) who used this term in the context of payment cards. Price coherence often arises from contractual restrictions that intermediaries impose implicitly or explicitly, notably prompting various litigation and regulatory interventions.[1]

In this article, we develop a model in which buyers and price-setting sellers choose whether to join an intermediary that can add some value to transactions between them. Unlike most platform models, the intermediary is not necessary for the transaction. Each buyer can purchase directly from the seller of its choice, or, if both buyer and seller have joined the intermediary, the buyer may choose to purchase through the intermediary. In such a setting, we consider what happens when the intermediary can first impose price coherence.

We find that price coherence leads to an inefficiency in that some buyers join the intermediary when, considering all costs, they should not. Thus, price coherence causes over-consumption of intermediaries' services and inhibits efficient disintermediation. We also find that price coherence results in inflated retail prices and lower consumer surplus. Indeed, in our setting consumer surplus ends up below the level that occurs without intermediation. Moreover, the intermediary chooses an excessive level of benefits to buyers. Greater buyer-side benefits increase both the demand from buyers and the fees the intermediary can charge sellers under price coherence, so an intermediary invests until the marginal dollar of investment yields less than a dollar of benefit to buyers. Even intense competition between intermediaries may not fix these distortions because price coherence suppresses competition between intermediaries on the basis of cost. Rather, with price coherence, we find that competition steers buyers towards the intermediary that offers the greatest buyer-side benefits, thereby magnifying these distortions.

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[1]For example, a recent decision rejects American Express "no steering" rules, and European regulatory inquiries and U.S. consumer litigation challenge contract provisions preventing hotels from offering lower prices to customers who book directly or through low-cost booking services. Section 2 provides further details.

Price coherence operates by making all buyers share the intermediary's fees to sellers, thereby raising the price of direct purchases. This increases demand for the intermediary's service, allowing it to charge more and increasing its profit. This mechanism operates through three related channels. First, when an intermediary charges sellers positive fees for intermediated transactions, price coherence results in all buyers sharing these fees through higher retail prices. This makes direct purchase (or purchase through another intermediary) more expensive, which increases the number of buyers joining and using the intermediary. Second, with price coherence in place, an intermediary does not face a reduction in demand when it raises seller fees, provided that sellers continue to participate. Finally, an intermediary can provide rebates or other benefits to buyers and correspondingly increase its fees to sellers, which further raises the cost of direct purchase and the benefits of intermediated purchases, thereby attracting even more buyers to join while ensuring that sellers remain willing to join.

To illustrate the theoretical mechanism driving our results, we start with a model in which buyers and price-setting sellers choose whether to join a single intermediary that can add some value to transactions between them. In this environment, we obtain our key result that intermediation with price coherence is excessive and reduces consumer surplus compared to the situation without price coherence, as well as compared to the situation without intermediation. The reduction in consumer surplus follows because, with price coherence, each seller is willing to pay the intermediary a fee that equals the expected per-buyer benefit that buyers receive from using the intermediary to buy from a seller since this allows the seller to increase its retail price by an equivalent amount without losing any demand. (This condition is known as merchant internalization in the payments literature.) Thus, the increase in retail prices all buyers face as a result of intermediation cancels out the extra benefits they obtain. Buyers are left with the extra costs they incur in joining the intermediary in the first place, implying that they are worse off in aggregate. In contrast, in the case without price coherence, buyers face the intermediary's fee directly, and a buyer only uses the intermediary if it increases her net surplus. Note, however, that the increase in retail prices caused by intermediation with price coherence is unlikely to be obvious to buyers. As a result, buyers may erroneously feel that they are better off with price coherence (which yields "free" intermediation benefits) and worse off without price coherence (due to explicit charges for the intermediary's service).

Buyers participate, even though they are jointly worse off from doing so, due to a coordination failure. An individual buyer faces the same high price whether or not she joins the intermediary. If buyers could coordinate, they would take into account the higher price that results from their individual decisions to join the intermediary, and collectively they would prefer not to join the intermediary. The

situation for sellers is more subtle. Each seller does not care that by joining the intermediary and thus agreeing to price coherence, it makes direct purchases less attractive and intermediated purchases more attractive. (Each seller only cares about what happens to its overall demand.) But this shift in relative benefits of the purchase channels is what makes buyers want to join the intermediary in the first place. In our model, sellers ends up with the same profit in equilibrium as they would obtain in the absence of intermediation. If the intermediary tried to charge sellers more, sellers would not join.

Price coherence leads to an additional distortion through excessive investment in buyer-side benefits. With price coherence, the costs of higher buyer-side benefits are shared by all buyers, including those who buy directly. We show that this causes the intermediary to over-invest in buyer-side benefits, which further raises demand for the intermediary, the fees charged to sellers, and retail prices. Despite the higher fees, sellers still participate because buyers get higher benefits which increase their willingness to pay. With higher demand from buyers and with sellers willing to pay for the benefits their buyers enjoy, an intermediary invests until the marginal dollar of investment yields less than a dollar of benefit to buyers.

We also consider the case in which multiple intermediaries compete. Price coherence raises the price of purchasing through intermediaries that do not impose it. The intermediaries that impose price coherence can thus attract demand from those that do not, raising fees to sellers to fund greater buyer-side benefits to attract more buyers. We show that, far from curing the problems, competition between intermediaries can cause the effects to become larger and to occur more broadly. For example, if two identical intermediaries compete in prices, competition can lead them to invest in buyer-side benefits to the point where no net benefits are created. We show that the same result holds under imperfect competition between intermediaries, in the limit as the number of intermediaries becomes large. These results show that our mechanism does not rely on the existence of a monopoly intermediary. However, the results do rely on competition between intermediaries working in a particular way: Each intermediary holds market power over sellers with respect to transacting with the buyers that the intermediary has attracted. This competitive bottleneck arises if buyers tend to rely on a single intermediary while sellers are willing to join multiple intermediaries to reach those buyers.

While our theory predicts a loss in consumer surplus from intermediation with price coherence, we note these results are driven by sellers being willing to pay higher fees commensurate with the higher benefits their buyers get from going through the intermediary. If sellers' willingness to pay these fees is lower than our theory predicts (perhaps because sellers do not fully account for buyers' benefits, or if sellers are otherwise prevented from raising prices), intermediation with price coherence could increase

consumer surplus, compared to the case without intermediation, even though consumer surplus could increase even more under intermediation without price coherence.

We proceed as follows. In Section 1.1 we survey the relevant literature. In Section 2 we present three markets with price coherence. Section 3 provides a formal model of the structure of affected markets, and we analyze this model in Section 4. Section 5 allows for identical competing intermediaries. We also consider how our findings are affected by imperfectly competing intermediaries (Section 6.1), usage heterogeneity (Section 6.2), membership fees (Section 6.3), and asymmetric sellers (Section 6.4). In Section 7 we discuss possible policy prescriptions.

1.1   Related Literature

A well-developed literature explores the microstructure of exchange between buyers and sellers, and develops the role of intermediaries (e.g. marketmakers or traders) in purchasing products from sellers and reselling them to buyers. Important contributions include Gehrig (1993), Spulber (1996), Rust and Hall (2009), and Antras and Costinot (2011). In contrast to our paper, these papers consider contexts in which intermediaries quote bid and ask prices so that sellers do not set prices to buyers directly, and the issue of price coherence does not arise.

Closer to our work is the burgeoning literature on multi-sided platforms, pioneered by Caillaud and Jullien (2003), Rochet and Tirole (2003), Parker and Alstyne (2005), and Armstrong (2006). In contrast to most of this literature, we model the micro structure of the interactions between buyers and sellers by modeling price-setting sellers that compete to offer a product to buyers. Hagiu (2009) and Belleflamme and Peitz (2010) are among the few papers modeling the micro-structure of buyer and seller interactions in multi-sided platforms, but they do not allow buyers to bypass the platform and purchase directly from sellers, which is critical for evaluating price coherence. Baye and Morgan (2001) and Galeotti and Moraga-González (2009) allow buyers to bypass the platform, but they assume the platform is essential for buyers to be able to choose between sellers: In their models, a buyer who bypasses the platform has no way to access competing sellers and thus faces a monopoly seller. In contrast, we allow sellers to compete for buyers both directly and through a platform.

Closest to our work is the literature modeling payment card systems, in which buyers can purchase from sellers using a payment card or cash. Like that literature, we consider the efficiency of fee structures that emerge with price coherence (i.e. that there is no surcharge for payment by card) and merchant internalization. Articles in the payment literature that share these features include Rochet

and Tirole (2002, 2011), Wright (2004, 2012), Farrell (2006), and Guthrie and Wright (2007). Unlike these papers, we focus on the welfare effects of price coherence, and we establish a distortion that arises on the extensive margin (whether buyers join a particular card system), whereas the existing literature focuses on the intensive margin (whether cards are efficiently used). Like Wright (2012) and Bedre-Defolie and Calvano (2013), our results indicate a systematic bias in the fee structure towards card users resulting in the excessive use of payment cards.

More broadly, the mechanism driving our results resembles the mechanism in the literature on "externalities on non-traders" (Segal, 1999). In this literature, a principal commits to publicly observed bilateral contracts with multiple traders, which can create inefficiencies due to externalities from traders on non-traders. With negative externalities on non-traders, there may be excessive trade from the social viewpoint. Price coherence functions similarly: as more buyers join M, sellers set higher prices, and non-traders are correspondingly worse off. However, the imposition of price coherence is not a special case of Segal (1999) which requires the homogeneity of agents, whereas buyers and sellers play fundamentally different roles in our analysis. Segal's framework also requires a wider contract space in which the principal uses monetary transfers to extract the surplus that each agent gets from trading, which makes it much easier for the intermediary to benefit from price coherence, as we show in Section 6.3.

Since price coherence entails intermediaries constraining suppliers' pricing decisions, it is properly viewed as a vertical restraint. However, price coherence differs significantly from widely-studied vertical restraints. One might compare price coherence to resale price maintenance (RPM) agreements. But RPMs restrict absolute price, whereas price coherence restricts relative price. Price coherence could be understood as a maximum RPM applied by the intermediary for its add-on service (specifically a maximum of zero), but the RPM literature does not consider this type of structure. One might also compare price coherence to most-favored-nation (MFN) rules, in which a seller agrees to treat a given buyer as well as its most-favored buyer. But MFNs apply *across* buyers (disallowing a lower price to certain buyers) whereas price coherence applies to a *given* buyer across different channels (disallowing a lower price through a different intermediary or directly). Specifically, a MFN does not prevent a seller from setting different prices to the same buyer for different services (e.g. paying cash versus credit), and price coherence does not stop a seller from price discriminating against buyers (as airlines commonly do).

Separately, a small literature examines the use of commissions and kickbacks by intermediaries, most prominently Inderst and Ottaviani (2012). We assume that all buyers are perfectly informed,

eliminating concerns about an intermediary steering buyers to less suitable sellers. Rather, our concern is that price coherence encourages a buyer to use the intermediary rather than purchasing directly, or a high-cost intermediary rather than a low-cost intermediary. This does not arise in Inderst and Ottaviani because they only consider a single intermediary and do not allow buyers to purchase directly from sellers.

Throughout our analysis, we assume that buyers are fully rational. A recent literature considers intermediation under consumer naiveté, in which buyers find it difficult to evaluate the complex products on offer (Inderst and Ottaviani, 2012) or misperceive some product attributes (Murooka, 2015). Notably, our model does not require any such naiveté. Our model could be extended to allow buyers to find intermediaries' benefits, rebates, or fees more salient than equal changes in sellers' base price, but such extensions are beyond the scope of our paper.

2 Markets with price coherence

We now turn to three specific markets with price coherence. For each, we identify the source of the price coherence constraint, whether imposed by intermediaries (explicitly or implicitly), governments (law or regulation), or otherwise. We then note applicable fees to sellers as well as benefits to buyers, including significant changes over time.

In Edelman and Wright (2015), we extend these examples and provide further details including how regulators, sellers, and alternative intermediaries have attempted to change the market structure; how the intermediaries launched; how price coherence gained traction; and how competition and regulatory authorities have responded. We also provide references for the factual claims in this section, and we discuss additional intermediated markets in which price coherence plays an important role. Table 1 lists ten such markets and notes the source of price coherence in each.

2.1 Credit and debit cards

Credit and debit cards facilitate all manner of purchases by both consumers and businesses. Prager et al. (2009) and Rysman and Wright (2012) present relevant institutions, incentives, and implications.

In general, gross prices are identical whether a buyer pays by credit card, debit card, or in some other way. In some jurisdictions, including ten U.S. states, laws disallow credit card surcharges. Visa and MasterCard used contracts to impose similar rules. That said, litigation and regulation have ended this restriction in some countries. For example, U.S. litigation required Visa and MasterCard

Table 1: Markets with Price Coherence

market	intermediary	direct alternative
credit and debit cards *	payment card platform	cash
travel booking networks *	Global Distribution System	airline call center and web site
online rebate services	Fatwallet and kin	direct access to merchant's site
hotel booking services *	online booking site	hotel web site or telephone
restaurant ordering *	Foodler, GrubHub, and kin	telephone ordering
restaurant reservations	OpenTable and kin	telephone reservations
marketplaces *	Amazon Marketplace and kin	direct purchase
insurance comparison services *	comparison website	direct purchase from insurer
insurance and financial advice	broker / advisor	direct purchase (where available)
ebook distribution *	Apple iBookstore	potential direct sales

Note: An asterisk denotes markets where intermediaries use explicit contracts to impose price coherence. For other markets, price coherence results from implicit contracts, law or regulation, or other factors.

to allow merchants to impose credit surcharges if they so choose, beginning in January 2013 (except where prohibited by state law). Though cash discounts and credit card surcharges are similar in their purpose, their effectiveness appears to differ significantly, and cash discounts are seldom used despite being generally permitted. (Bourguignon *et al.* (2014) helps explain their differing effectiveness and usage.)

To encourage consumers to join a given payment card and to shift spending to that card, payment card issuers offer significant benefits to consumers. Early credit cards offered delayed payment and various consumer protections, but no rebates. In 1986, Discover began to offer a 1% rebate card, and multiple Visa issuers added a similar benefit in 1994. Greater rebates became available later, now including multiple U.S. cards with comprehensive 2% rebates. Though the rebates flow through the multi-party card network structure, merchants' payments are the ultimate source of the rebated funds. Critics allege that this fee structure, which requires merchants to pay high fees and rewards cardholders for use, promotes over-use of credit cards (as well as some debit cards).

2.2 Travel booking networks

Global distribution systems (GDSs) connect airline reservation systems to travel agents (TAs). With hundreds of airlines and thousands of TAs, it would be burdensome to connect each airline to each TA. Instead, a few large GDSs (currently three: Amadeus, SABRE, and Travelport) broker the connections. The resulting structure typically has four parties: *Airlines* sell through *GDSs* to reach *TAs* which serve *travelers*. In the three-party framework of our model, TAs represent the agents which let buyers (travelers) access the intermediary (GDS).

TA multihoming costs are high: Changing to a new GDS requires new training and processes for TA staff, and connecting to multiple GDSs requires systems that are not widely available to combine their results. Thus, each TA is effectively limited to a single GDS. In order to reach business travelers who tend to buy the most expensive tickets, airlines need to connect to the GDSs used by the TAs chosen by those business travelers. Because each TA uses only a single GDS, an airline needs to appear in all GDSs if it wants all TAs to be able to sell its flights.

Changing regulations shape airlines' dealings with GDSs. Through 2003, if an airline owned a GDS, it was required to submit its fares and schedules to all GDSs—assuring price coherence. But by the end of 2002, all airlines had sold their interests in GDSs. Many airlines began to offer their lowest prices as "web fares" available only on their own web sites, to the dismay of TAs who sought to sell all fares. In subsequent negotiations, GDSs obtained "full content access" to all of an airline's fares in exchange for sharply lowering their fees to airlines. This contractual commitment restored price coherence, meaning that the base price of a ticket is the same whether the ticket is purchased directly from an airline versus from a TA. (Most TAs, like most airlines, now charge additional fees for tickets booked by phone.)

Save for switching costs, GDSs are largely interchangeable to TAs, so a TA typically chooses a GDS based on incentive payments. Historically, GDSs provided TAs with computer terminals and telecommunications links without charge—major benefits when IT was costly. Today, GDSs provide TAs with payments which often exceed $1.50 per flight segment. GDSs fund these payments to TAs by charging fees to airlines. GDS fees are confidential but are understood to be approximately $3 per segment, hence $12 for a domestic connecting round-trip. As airlines' GDS contracts came up for renewal, GDSs sought to raise the fees. By 2012, GDS fees met or exceeded prior levels. GDS payments to TAs have increased in parallel.

2.3 Rebate services

Online "cashback" rebate services offer users discounts when they purchase from participating e-retailers. A registered user clicks from a rebate service site to a merchant's site, makes a purchase from the merchant, and earns a rebate, often 5% to 10% paid after 30 to 90 days. Initially known only to the savviest shoppers, rebate sites have become mainstream: Alexa ranks Ebates the 338th most popular site in the U.S. as of March 2015 (about as popular as marriott.com). From a consumer's perspective, these rebates appear to be a windfall.

Multiple factors ensure equal prices for rebate service users. First, sites largely lack the ability to present different prices to users coming from different sources. In principle sites could add this function, albeit with some technical and billing complexity. An additional impediment is that users would view differing prices as improper. Indeed, users occasionally complain about higher prices when referred by a rebate service, although these have been glitches rather than systematic differences. Finally, rebate sites would not allow prices to differ: The CEO of a leading rebate service told one of the authors that he would ban a merchant that increases prices for rebate service users.

Rebate services differ from the other examples we examine in that they provide buyers with *money* rather than some other good or service. Ordinarily, intermediary benefits can create gains from trade— benefits worth more to buyers than the intermediary's cost of providing the benefits. In contrast, no gains from trade are possible when an intermediary provides money. Nonetheless, our model still applies in the case of pure rebates, provided the intermediary has some way to benefit from the increased participation of buyers due to price coherence. For example, the intermediary could charge a membership fee or sell advertising and cross-sell products to affiliated buyers.[2]

3 Model

We start with a model of a monopoly intermediary, M. There is a continuum (measure one) of buyers. There are multiple sellers, and each buyer wants to buy one unit of a product from one of the sellers. A buyer can buy the product directly from a seller or through M.

Intermediation costs and benefits

We consider intermediaries that provide benefits to buyers, such as offering complementary products, reducing transaction costs, and offering financial rebates. Suppose that by investing $k \geq 0$ per unit, M can provide a benefit that buyers value at $b(k)$ per unit.[3] Assume $b(k)$ is twice continuously differentiable with $b'(k) \geq 0$ and $b''(k) < 0$. Define $\pi(k) \equiv b(k) - k$, the net benefit generated by M, and assume $\pi'(0) > 0$. Benefits can be costly to produce in the sense that for sufficiently large investments in buyer-side benefits, M incurs a cost of more than one dollar to increase buyer-side benefits by an amount that buyers value at one dollar. In the case of rebates, this requires that the

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[2]A prior working paper version of our article (Edelman and Wright, 2014) offered a formal treatment of this environment.

[3]Our main findings continue to hold even if k is a fixed investment cost rather than a variable investment cost. See Online Appendix Section A.

cost of offering an additional dollar of rebate exceeds one dollar for sufficiently large rebates. We implement this assumption by requiring that there exists some sufficiently large investment $\overline{k} > 0$ such that $\pi(\overline{k}) = 0$ and $\pi'(\overline{k}) < 0$. Define k^e as the efficient level of investment, which maximizes $\pi(k)$. This clearly exists and is unique given our assumptions, and is defined by $\pi'(k^e) = 0$. Note that $0 < k^e < \overline{k}$. Define $\pi^e \equiv \pi(k^e)$, the maximum net benefit the intermediary can generate. Our assumptions imply $\pi^e > 0$.

From an individual buyer's perspective, the costs of joining M may be significant. These may include the costs of filling out forms, finding and contacting the intermediary, evaluating an intermediary's offer, and learning to use the intermediary's service. We therefore suppose each buyer draws a joining cost c from the distribution G over $[0, \overline{c}]$ with corresponding density g. Over this interval, assume G is strictly increasing, twice continuously differentiable, and log-concave. We assume $\overline{c} > b(\overline{k})$. This ensures that not all buyers join the intermediary in equilibrium, even if it imposes price coherence and invests in buyer-side benefits until there is no *net* benefit. Note that c is a sunk cost: After a buyer incurs c, it does not distort the buyer's decision to use M versus purchase directly. The cost c provides a convenient way to capture that the demand for M is elastic and that there is an extensive margin over which our mechanism can apply.[4] The cost c also captures the idea that, while buyers obtain surplus from M, some of the surplus may be dissipated. In our model, surplus dissipates not only due to the cost c incurred by buyers joining M who should not, but also due to M's possible over-investment in benefits required to attract buyers due to the cost c.

Assume that sellers receive no direct benefit from using M. Instead, by joining M, a seller receives the benefit that M delivers to the seller's buyers. That is, a seller's (indirect) benefits are derived endogenously. We also assume that sellers do not incur joining costs when signing up for M's service. This reflects that such costs are normally a small part of the overall costs and revenues that a seller considers when joining M.

Seller competition

To model seller competition, assume that there are $n \geq 2$ symmetric and differentiated price-setting sellers. Each seller has a cost d per unit sold. Each buyer wants to buy one unit of a product, which she values at v, from one of the sellers. Assume that v is sufficiently large so that all buyers purchase one unit from one of these sellers in equilibrium (so the market always remains covered).[5]

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[4]In Section 6.2, we consider an extension in which buyers are heterogeneous in their valuations of *using* the intermediary's service rather than with respect to adoption.

[5]In Online Appendix Section B we provide a sufficient condition on v.

Let p_i^d be seller i's price for direct purchases, p_i^m be seller i's price for intermediated purchases, and p_B be the per-transaction fee charged by M to buyers for intermediated transactions. Then define $w_i = v - p_i^d + \max\{0, b(k) - p_B - (p_i^m - p_i^d)\}$ if the buyer and seller i have joined M, and $w_i = v - p_i^d$ otherwise. Note w_i represents a buyer's utility from the purchase of one unit from seller i, ignoring any disutility a buyer may face from having to buy from a seller that is located away from its ideal location in product space (i.e. mismatch cost). Assume all other sellers ($j \neq i$) set identical prices and make identical joining decisions, so offer the same level of utility which we denote by the scalar $\widehat{w}_{-i}$. Assume that facing these utility levels, the share of buyers that choose seller i is given by

$$s_i(w_i, \widehat{w}_{-i}) = \frac{1}{n} + \sigma(w_i - \widehat{w}_{-i}). \tag{1}$$

Here σ is a constant parameter for any given n. Let $t = \frac{1}{\sigma n}$ be the seller markup that arises in the symmetric equilibrium that follows from this specification in the absence of any intermediary. (See the proof of Proposition 1.) If $n = 2$, then t corresponds to the transportation cost parameter in the standard Hotelling demand specification. With different values of σ and allowing for $n > 2$, this model is consistent with the demand specification arising from the Salop (1979) circular city model, the pyramid model of Ungern-Sternberg (1991), and the spokes model of Chen and Riordan (2007), among others.[6] Because the market is assumed to be fully covered, and because we focus on symmetric equilibria, the mismatch or transportation costs implicit in this setup do not affect our welfare results. However, we assume that the equilibrium markup t is sufficiently large that no seller would want to set prices in a way that captures all buyers of a given type (e.g. all those that have joined M or all those that have not joined M). The condition $t > b(\overline{k})$ suffices.

We assume the linear structure in (1) in order to ensure that merchant internalization established in the payments literature (e.g. Rochet and Tirole 2002 and 2011 and Wright 2004 and 2012, among others) also holds in our setting. This condition says that the fee that sellers are willing to pay to join M in equilibrium will be exactly equal to the net benefit buyers receive from being able to use M to buy from a seller. The linear form of demand allows us to aggregate across buyers that join the intermediary and those that do not, and still obtain the merchant internalization result. This allows us to obtain a closed form solution for the fees charged to sellers under price coherence (i.e. $p_S = b(k) - p_B$). In general, there is no closed form solution for the fee sellers are willing to pay once we relax the linear structure in (1) given that sellers face a mix of buyers.[7] However, Farrell (2006)

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[6]The generality implicit in (1) is possible because we focus on symmetric equilibria. To characterize any symmetric equilibrium, we only need to consider the deviation of one seller, with all other sellers making identical decisions.

[7]In the special case that all buyers join the intermediary, Ding (2014) shows merchant internalization holds very

shows that merchant internalization still holds as an approximation in general, which suggests that the linear structure in (1) does not drive our results.

The intermediary's instruments

Where indicated, we allow M to impose *price coherence*, i.e. to require that any seller participating in M must set the same price for buyers that reach the seller through M as for buyers that purchase directly from the seller.

Throughout, we restrict attention to linear non-negative (per-transaction) fees, denoted p_B to buyers and p_S to sellers. In contrast, a negative p_B would imply that M can offer costless rebates (incurring only a cost equal to the amount of the rebate). Then M could impose price coherence and set p_B sufficiently negative to attract all buyers to join. As we show in Online Appendix Section C, the results on excessive intermediation and consumer surplus in Section 4.3 continue to hold in this case if demand is given by (1), as well as in the case with Shubik and Levitan (1980) demand. (This includes the case of a monopoly seller facing linear demand.) However, there would no longer be any excessive investment in buyer-side benefits.

A notable implication of our focus on linear fees is that M cannot charge membership fees to the buyers who wish to join M. With price coherence, a monopoly intermediary would want to charge such fees in order to capture some of the surplus it would otherwise leave with buyers. We explore this possibility, which generally makes it easier to establish our results for a monopoly intermediary (e.g. on the loss of consumer surplus due to price coherence) in Section 6.3. We do not focus on this case because, in practice, few intermediaries charge such fees. (Annual fees for some payment cards are a notable exception.) One reason such fees are not used is that they may be competed away once we introduce competition between intermediaries, as shown in Section 6.3. Another possibility is that membership fees make it more difficult for an intermediary facing a chicken-and-egg problem to launch.

Timing

The timing is:

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generally. For this reason, our key results can also be obtained with a general model of seller competition that allows for elastic aggregate demand, provided that in the equilibrium with price coherence, M attracts all buyers to join. The results can also be obtained with a general linear demand specification with elastic demand as in Shubik and Levitan (1980). See Online Appendix Section C.

1. M determines the per-unit fees p_B and p_S it charges to buyers and sellers, and whether to impose price coherence.

2. Each buyer observes its value of c and decides whether to join M. If a buyer joins M, the buyer incurs the cost c. Each seller decides whether to join M and what price(s) to set to buyers.

3. Buyers draw mismatch values for each seller and choose a seller to purchase from. If both the buyer and its chosen seller have joined M, the buyer decides whether to complete the purchase through M. All benefits, costs and transfers between parties are realized.[8]

The assumption that buyers and sellers make their joining decisions at the same time is not innocuous. If sellers get to decide first, a seller could consider not joining M in order to dissuade buyers from joining M in the first place, instead attracting these buyers with its lower prices. However, such a strategy will not work if buyers do not consider the particular seller's policy when deciding whether to join M. Moreover, even if buyers do consider such a seller's policy, a buyer may not yet know whether she will want to purchase from that seller, so the buyer may still want to join M. In that case, the distortions predicted by our model still arise, albeit to a lesser extent reflecting the likelihood of a given buyer wanting to purchase from this seller. (See Section 6.4.)

Equilibrium definition

Our equilibrium concept is subgame perfect equilibrium. Due to the fixed joining cost buyers face, there are typically multiple equilibria in the continuation game following given fees. For instance, there always exists a trivial equilibrium in the continuation game in which M does not attract any trade because buyers do not expect sellers to join and vice versa. To select among equilibria in the continuation game, we assume that if there are equilibria in which M can profitably attract one or more of the sellers, then one of these is selected. This captures the idea that M should be able to overcome the trivial equilibrium if it is profitable to do so.[9] In light of our symmetric demand specification in (1), we restrict attention to equilibria in which all sellers set identical prices and make identical joining decisions.[10]

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[8]Specifically, the benefits v, mismatch costs and the price paid from buyer to seller are realized for completed purchases. In addition, the benefits $b(k)$, costs k, and fees p_B and p_S are realized on purchases through M.

[9]Hagiu and Eisenmann (2007) and Hagiu and Spulber (2013) discuss mechanisms by which intermediaries can coordinate buyers and sellers on equilibria that entail use of the intermediary.

[10]In Section 6.4, we consider an asymmetric setting in which not all sellers join M.

4 Monopoly intermediary

In this section, we analyze the model introduced in Section 3. We first consider what happens without price coherence (Section 4.1) and with price coherence (Section 4.2), then evaluate the impact of M imposing price coherence (Section 4.3).

4.1 Intermediation without price coherence

Without price coherence, sellers pass through to buyers any fees they are charged by M, so a buyer is willing to join M provided the net benefit the buyer obtains, $b(k) - p_B - p_S$, exceeds the buyer's joining cost c. The number of buyers wanting to join and use M is $G(b(k) - p_B - p_S)$. With the ability to charge more for intermediated transactions, sellers always join M. To maximize its profit, M sets p_B and p_S to maximize $(p_B + p_S - k)G(b(k) - p_B - p_S)$. Since profit depends only on $p_B + p_S$, we set $p_B = 0$ without loss of generality.

M chooses the efficient investment level k^e that maximizes the net benefit $b(k) - k$. This occurs where the marginal benefit to buyers of one extra dollar of investment exactly equals one dollar. To see why this is profit maximizing for M, note that if $k < k^e$, then M can increase k and p_S by the same amount, and enjoy the same margin but higher demand, since $b'(k) > 1$. By the same logic, M can increase its profit if $k > k^e$ by decreasing k and p_S. M's fee reflects the usual monopoly trade-off between a higher margin per transaction and a reduced number of transactions as fewer buyers join. The resulting fee, p_S^m, follows the standard monopoly pricing formula

$$p_S^m = k^e + \frac{G(b(k^e) - p_S^m)}{g(b(k^e) - p_S^m)}, \tag{2}$$

where k^e is M's marginal cost, and $G(b(k) - p_S)$ is the demand faced by M, which is decreasing in its price p_S. Proposition 1 formalizes these results.

Proposition 1 *Suppose M cannot impose price coherence. There exists an equilibrium in which M invests $k = k^e$ in benefits to buyers and sets the fees $p_B = 0$ and $p_S = p_S^m$, where p_S^m is the solution to (2) and satisfies $k^e < p_S^m < b(k^e)$. Sellers join M and set the equilibrium price $d + t$ for buyers that purchase directly and the price $d + p_S^m + t$ for buyers that purchase through M. Buyers join if and only if their joining cost satisfies $c \leq b(k^e) - p_S^m$.*

The appendix provides the proof of this result and all others.

The equilibrium in Proposition 1 entails sellers passing through to their buyers any fee they are charged by M, so M does just as well by charging sellers as by charging buyers directly. The equilibrium

in Proposition 1 is equivalent to other equilibria with $p_B \geq 0$ and $p_S \geq 0$ such that $p_B + p_S = p_S^m$, in that all such equilibria yield identical joining decisions by buyers and sellers, as well as identical amounts paid or received by each agent. This equivalence is consistent with the more general neutrality result of Gans and King (2003). The only symmetric equilibrium not in this unique equivalence class is the trivial equilibrium in which no buyers or sellers join, which is ruled out by our equilibrium selection criterion.

4.2 Intermediation with price coherence

As noted in the introduction, there are three separate but related channels by which M benefits from imposing price coherence.

First, imposing price coherence but holding M's other choices at the same levels as in the equilibrium without price coherence, buyers become more willing to join M. To see this, note that with price coherence, a buyer is willing to join M provided that the benefits the buyer obtains, $b(k^e)$, exceed the buyer's joining cost c. As a result, with price coherence, the number of buyers joining and using M is $G(b(k^e))$ rather than $G(b(k^e) - p_S^m)$ without price coherence. The fees sellers face, p_S^m, are shared among all buyers, resulting in buyers paying $p_S^m G(b(k^e))$ more for direct purchases and $p_S^m (1 - G(b(k^e)))$ less for intermediated purchases.

Second, M benefits from price coherence because it can increase its fee to sellers without losing any demand. This is because the fee to sellers, p_S, no longer enters the demand that the intermediary faces from buyers, since sellers cannot pass this fee on to only those buyers that come through M. Instead, seller participation determines how much M can charge. Each seller is willing to join if the additional net benefit its buyers get from intermediated transactions, $b(k^e)$, is at least as large as the fee it faces, p_S. That is, merchant internalization determines how much sellers are willing to pay. Since $p_S^m < b(k^e)$, M can increase p_S up to $b(k^e)$ without facing any reduction in demand. It does best setting $p_S = b(k^e)$ to obtain a profit of $(b(k^e) - k^e)G(b(k^e))$.

Third, M further benefits from price coherence because it can invest more in benefits to buyers and correspondingly increase its fees to sellers—thereby attracting even more buyers to join while ensuring that sellers remain willing to join. To see this, notice that the preceding analysis holds for any k. Therefore, M's profit-maximizing choice of k is given by:

$$k^* = \arg\max_k \{(b(k) - k)G(b(k))\}. \tag{3}$$

M invests in buyer-side benefits beyond the efficient level k^e in order to expand demand $G(b(k))$.

Because merchant internalization applies, sellers are willing to pay the higher fee $p_S = b(k^*)$.

Proposition 2 formalizes the resulting equilibrium.[11]

Proposition 2 *Suppose price coherence is imposed. There exists an equilibrium in which M invests k^* satisfying $k^e < k^* < \overline{k}$ in creating benefits for buyers and sets the fees $p_B = 0$ and $p_S = b(k^*)$. Sellers join the intermediary and set the equilibrium price $d + b(k^*)G(b(k^*)) + t$ that applies for all buyers. Buyers join if and only if their joining cost satisfies $c \leq b(k^*)$.*

Note that setting $p_B > 0$ is never optimal since it would reduce the amount M can charge sellers by the same amount and would also lower demand from buyers to join M. Given $p_B \geq 0$, maximum profit is achieved by setting $p_B = 0$. With price coherence, sellers set their usual equilibrium prices based on marginal cost plus the constant markup t. The only change is that sellers' marginal costs increase from d to $d + b(k^*)G(b(k^*))$. The additional term reflects the fee they pay to M per transaction, which is $b(k^*)$, multiplied by the proportion of their transactions which are through M, which is $G(b(k^*))$.

4.3   Impact of price coherence

A comparison of the equilibrium outcomes with and without price coherence makes it clear that M wants to impose price coherence. With price coherence, M obtains $\max_k\{(b(k) - k)G(b(k))\}$ rather than the lower amount $\max_{k,p_S \geq 0}\{(p_S - k)G(b(k) - p_S)\}$ that arises without price coherence. The comparison in profit expressions highlights that M's demand curve shifts up with price coherence, allowing M to charge higher fees to sellers and at the same time attract more buyers.

Buyers are worse off in aggregate with price coherence compared to the case in which M does not exist. Compared to the case without M, the price of goods ends up higher by $b(k^*)G(b(k^*))$, which represents a loss in surplus for all buyers. On the other hand, $G(b(k^*))$ buyers purchase through M and get the benefit $b(k^*)$. These two effects exactly cancel out, and the net effect of M on buyers is that buyers incur an additional cost of joining that equals $E[c|c < b(k^*)]G(b(k^*))$, where E is the expectation operator. While intermediation with price coherence makes buyers worse off in aggregate, there are distributional effects. Among those joining M, some are better off (those with $c < b(k^*)(1 - G(b(k^*)))$) and some are worse off (those with higher c). On the other hand, all the buyers not joining M are worse off—they obtain no benefit from intermediation despite facing increased prices. This loss in consumer

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[11] As we show in the proof of the proposition, the equilibrium characterized in the proposition is the unique symmetric equilibrium, other than the trivial equilibrium in which no buyers and sellers join, which is ruled out by our equilibrium selection criterion.

surplus caused by intermediation with price coherence contrasts with the positive surplus M creates without price coherence for all buyers that use its services. If sellers for some reason do not take into account the full benefit $b(k^*)$, intermediation with price coherence could increase consumer surplus (compared to the case without intermediation) even though intermediation without price coherence could increase consumer surplus even more.

An efficient outcome entails M choosing the efficient level of investment k^e and buyers facing a fee of k^e to use M's service, so that buyers would face the true cost of the service they consume. This would ensure that the marginal buyer contributes exactly zero to welfare, and welfare would match the first-best solution. Compared to this ideal, both the case without price coherence and the case with price coherence involve distortions.

Without price coherence, while M invests efficiently in buyer-side benefits, buyers end up paying the monopoly fee p_S^* to use M's services, which is more than k^e. As a result, all buyers (even the marginal buyer) make a positive contribution to welfare. Due to M's market power, too few buyers join M compared to the efficient level.

In comparison, with price coherence in place, buyers do not face the cost of providing M's service. As a result, too many buyers join. Moreover, M invests excessively in buyer-side benefits. Both of these effects result in too many buyers completing their purchases through M compared to the first-best level, with some inframarginal buyers joining M and making a negative contribution to welfare.

Proposition 3 summarizes these results.

Proposition 3 *M always imposes price coherence if permitted to do so. M chooses the efficient investment in buyer-side benefits without price coherence, but over-invests in creating these benefits with price coherence. Additionally, too few buyers join M without price coherence, and too many buyers join M with price coherence. Moreover, consumer surplus increases when M operates without price coherence but decreases when M operates with price coherence, compared to the case in which M does not operate at all.*

If buyers could coordinate, they would be jointly better off not joining M. Thus, although intermediation with price coherence seems like a windfall from an individual buyer's perspective, buyers actually end up worse off, taking into account higher seller prices. Sellers end up unaffected by price coherence. This reflects that each seller is willing to pay a fee to M equal to the amount by which it can increase its price without losing demand. This in turn equals the buyers' benefit of using M (i.e. $b(k^*)$ in equilibrium).

Reflecting that too few buyers join M without price coherence and too many buyers join M with price coherence, the effect of price coherence (or the existence of M) on total welfare is ambiguous in this setting. Explicit conclusions on total welfare are possible only for special cases. For example, if $G(c)$ is the power function $G(c) = (\frac{c}{\bar{c}})^\alpha$, where $\alpha > 0$, then M's contribution to welfare under price coherence is $(b(k^*) - k^* - E[c|c < b(k^*)])G(b(k^*)) = \left(\frac{b(k^*)}{\bar{c}}\right)^\alpha \left(\frac{1}{1+\alpha}b(k^*) - k^*\right)$. This is negative if and only if $\frac{b(k^*)-k^*}{k^*} < \alpha$. In this case, eliminating price coherence (or M) would increase welfare. Here α is the elasticity of the demand function faced by the intermediary with respect to the benefits it offers to buyers $b(k)$. The result indicates that the intermediary destroys welfare under price coherence if the buyer-side benefits created by M are insufficiently valuable relative to M's cost in providing these benefits. For example, if demand is linear (i.e. $\alpha = 1$), the intermediary destroys welfare if and only if the rate of return on investment is less than 100%. If demand is more elastic with respect to the benefits offered to buyers, then an even greater return on investment would be required to avoid M destroying welfare. This result indicates that an intermediary is especially likely to reduce welfare if it creates limited net value in the first place and if buyers' demand for M is quite elastic with respect to the benefits offered to buyers. More general welfare results are possible when intermediaries compete head-to-head, which is the case we turn to next.

5   Competing intermediaries

One might hope that the entry of a rival intermediary would help address the distortion in fees that arises when a monopoly intermediary imposes price coherence. Our analysis suggests otherwise.

Consider the incentives of buyers. To attract buyers away from an established intermediary, an entering intermediary needs to offer greater benefits to buyers. This reflects that, facing non-trivial joining costs, buyers tend to join a single intermediary, while sellers are willing to join multiple competing intermediaries to cater to those buyers. As a result, entry tends to push intermediaries to compete to attract buyers, accentuating buyer-side benefits.

As in the case of a single intermediary, three related channels drive these results. First, when some intermediary M^1 charges sellers higher fees than are charged by competing intermediaries, price coherence ensures that the buyers that choose rival intermediaries share those higher fees. Thus, purchases through other intermediaries become more expensive, increasing the demand for M^1. Second, with price coherence, an intermediary does not face a reduction in demand as it raises its fees to sellers, provided that sellers continue to participate. Third, by raising its fees to sellers and raising the

benefits it offers buyers, an intermediary attracts buyers away from other intermediaries with lower benefits—even though those intermediaries have lower fees to sellers. As a result, excessive fees and benefits persist in equilibrium, and too many buyers join intermediaries.

In this section, we formally model competition between two identical intermediaries to capture this logic. (The case with two or more differentiated intermediaries is handled in Section 6.1.) Each intermediary decides whether to impose price coherence, and sets its fees and investment level k in stage one. Each buyer must incur a cost c to join an intermediary in stage two, where each buyer draws c from G as before. If a buyer joins both intermediaries, it incurs its c twice, but can only buy through one intermediary when it ultimately makes a purchase. Each seller also decides whether to join one or both intermediaries in stage 2. The model is otherwise unchanged from the model of Section 4.

In this setting, it is natural for each seller to join both intermediaries and for each buyer to join only one. This leads to a competitive bottleneck equilibrium similar to that analyzed in Armstrong and Wright (2007). In the competitive bottleneck equilibrium, intermediaries compete to sign up buyers exclusively. Sellers want to access these buyers, and each intermediary then sets fees to sellers as if the intermediary does not face competition. The need to attract buyers to their respective service leads to even greater over-investment in buyer-side benefits, beyond what a monopoly intermediary would choose. This strengthens our previous results.

As in much of the literature on competing multi-sided platforms (e.g. Caillaud and Jullien (2003), Armstrong and Wright (2007), and Ambrus and Argenziano (2009)), there are often multiple equilibria in the continuation game following given fees and investment levels set by the two intermediaries. (For example, facing the same set of fees, all participating buyers and sellers could join one intermediary, or all could join the other.) To focus on the symmetric competitive bottleneck equilibrium, we assume that whenever there are equilibria in the continuation game in which all sellers join both intermediaries, then one such equilibrium is selected.

5.1 Intermediation without price coherence

We first consider what would happen without price coherence. Denote the two intermediaries M^1 and M^2. Define $\beta^i = b(k^i) - p_B^i - p_S^i$, which measures the surplus M^i provides to a buyer and seller in a single transaction. Given that the fee to sellers p_S is passed on to buyers, buyers join the intermediary M^i offering the highest surplus β^i, provided that this is positive. Since M^i can take the whole market

rather than half the market if it increases β^i above β^j by an infinitesimal amount, it always wants to do so provided that its margin remains positive. As a result, in equilibrium, M^i offers the maximum possible surplus β^i consistent with cost recovery (i.e. $p_B^i + p_S^i \geq k^i$). This is achieved by setting $k^i = k^e$ and $p_B^i + p_S^i = k^e$ for $i = 1, 2$. Bertrand-like competition between intermediaries ensures that each buyer faces the cost k^e of using an intermediary's service, either directly or through a seller's prices, and therefore each buyer joins and uses an intermediary's service only when it is efficient to do so.

Proposition 4 *Suppose there are two identical competing intermediaries that cannot impose price coherence. In the symmetric competitive bottleneck equilibrium, both intermediaries invest $k = k^e$ in benefits to buyers and set the fees $p_B = 0$ and $p_S = k^e$. Sellers join both intermediaries and set the equilibrium price $d+t$ for buyers that purchase directly and the price $d+k^e+t$ for buyers that purchase through an intermediary. Buyers with joining cost $c \leq \pi^e$ randomize over which intermediary to join and use. Buyers with $c > \pi^e$ do not join either intermediary.*

Proposition 4 establishes that if intermediaries cannot impose price coherence, they invest efficiently in buyer-side benefits (i.e. choosing k^e), and pass the associated costs on to sellers who in turn pass them on to buyers. This leads to the first-best outcome, with buyers rather than the intermediaries retaining the full surplus.

However, this outcome is no longer an equilibrium satisfying our equilibrium selection rule if intermediaries can impose price coherence in stage 1. Suppose M^1 deviates and imposes price coherence in stage 1. It can continue to invest k^e, charge sellers a fee just above k^e, and charge nothing to buyers. Sellers that join M^1 must set a single price for all buyers, implying that buyers strictly prefer to join M^1 over M^2 if sellers join both intermediaries. Such buyers get the same benefit $b(k^e)$ but avoid the extra fee from sellers. With $G(b(k^e))$ buyers expected to join M^1 exclusively, sellers are willing to join M^1 as shown in the proof of Proposition 2, and M^1 makes a positive profit. Thus, each intermediary prefers to impose price coherence if the other does not, ruling out the efficient outcome.

5.2 Intermediation with price coherence

Next we show that there is a competitive bottleneck equilibrium in which both intermediaries impose price coherence. When both intermediaries impose price coherence, buyers join the intermediary M^i offering them the highest surplus $b(k^i) - p_B^i$ subject to $\beta^i \geq 0$ which ensures that sellers want to join the intermediary (i.e. $p_S^i \leq b(k^i) - p_B^i$ given that merchant internalization applies). Since M^i can

take the whole market rather than half the market if it increases $b(k^i) - p_B^i$ above $b(k^j) - p_B^j$ by an infinitesimal amount, it always wants to do so provided that its margin remains positive. As a result, in equilibrium, M^i offers buyers the maximum possible surplus $b(k^i) - p_B^i$ consistent with cost recovery (i.e. $p_B^i + p_S^i \geq k^i$) and $\beta^i \geq 0$. This is achieved by setting $k^i = \overline{k}$, $p_B^i = 0$ and $p_S^i = b(\overline{k})$.

Proposition 5 *Suppose there are two identical competing intermediaries that can impose price coherence. In the symmetric competitive bottleneck equilibrium, both intermediaries adopt price coherence. The intermediaries invest $k = \overline{k}$ and set fees $p_B = 0$ and $p_S = b(\overline{k})$. Note $\overline{k} > k^e$ which implies excessive investment in buyer-side benefits. Buyers with joining cost $c \leq b(\overline{k})$ randomize over which intermediary to join and use. Buyers with $c > b(\overline{k})$, do not join either intermediary. Sellers join both intermediaries and set the equilibrium price $d + b(\overline{k})G(b(\overline{k})) + t$ that applies for all buyers.*

Compared to the case with a single intermediary, competition between rival intermediaries works to further raise their investment in buyer-side benefits in order to attract buyers to their respective services. Specifically, in the competitive bottleneck equilibrium, Bertrand-like competition between intermediaries means that they continue to attract buyers until the benefit they create per transaction, $b(k)$, equals the cost to them of creating this benefit, k. This is the highest level of benefit that intermediaries can offer buyers while still breaking even. This equality occurs at the investment level $\overline{k}$.

5.3 Impact of price coherence

Without price coherence, investment in buyer-side benefits is efficient ($k = k^e$) and the first-best number of buyers join (i.e. those with $c \leq \pi^e$). With price coherence, there is excessive investment in buyer-side benefits ($k = \overline{k}$) and far too many buyers join (i.e. those with $c \leq b(\overline{k})$, where $b(\overline{k}) > b(k^e) > \pi^e$).

The intermediaries make zero profit with or without price coherence, reflecting that they are not differentiated. The intermediaries' contribution to welfare is therefore always equal to their contribution to consumer surplus. Without price coherence, the intermediaries' contribution to consumer surplus and welfare is $\int_0^{\pi^e} (\pi^e - c)dG(c)$. With price coherence, the intermediaries no longer generate any net benefit, and their contribution is $-\int_0^{b(\overline{k})} c\,dG(c)$. The loss in consumer surplus and welfare due to price coherence applies even if most buyers have very low values of c. Proposition 6 summarizes these results.

Proposition 6 *Suppose there are two identical competing intermediaries. There is efficient investment in buyer-side benefits without price coherence, but over-investment in creating these benefits with price coherence. Additionally, the first-best number of buyers join intermediaries without price coherence, but too many buyers join intermediaries with price coherence. Moreover, eliminating price coherence increases consumer surplus and welfare by $\pi^e G(\pi^e) + \int_{\pi^e}^{b(\overline{k})} c\, dG(c)$. Eliminating intermediation increases consumer surplus and welfare by $\int_0^{b(\overline{k})} c\, dG(c)$.*

The market failure underlying the results in Proposition 6 results from a coordination failure by buyers, in light of the incentives suppressed by price coherence. Intermediaries need not benefit from this market failure, and this market failure can arise even if intermediaries compete aggressively for buyers, as we assumed in this section.

6   Extensions

In this section, we discuss the implications of some modifications to our setup, showing that our main insights are generally robust although some details change. Formal proofs of all results in this section are in the Online Appendix.

6.1   Imperfect competition between intermediaries

In this subsection, we explore what happens when competing intermediaries are differentiated. We allow that there are $N \geq 2$ symmetric but differentiated intermediaries that compete, denoted $M^1, M^2, ..., M^N$. We use the Perloff and Salop (1985) model to capture differentiation between intermediaries. Each buyer incurs a cost c_i of joining M^i, where c_i is i.i.d. across intermediaries from the same distribution $G(c)$ with $c \in [0, \overline{c}]$. This contrasts to the model in Section 5, where c_i was perfectly correlated across intermediaries, so that intermediaries were identical from the perspective of buyers. The rest of the model remains identical to that in Section 5. Consistent with each buyer facing some cost to join each intermediary, we continue to focus on the symmetric competitive bottleneck equilibrium in which buyers join a single intermediary and sellers join all.

Without price coherence, buyers join M^i if $\beta^i - c_i \geq \max\{0, \max_{j\neq i}\{\beta^j - c_j\}\}$. The demand for M^i is therefore

$$D^i = \int_0^{\beta^i} \Pi_{j=1,j\neq i}^N (1 - G(\beta^j - \beta^i + c_i)) dG(c_i)$$

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and M^i's profit is $(p_B^i + p_S^i - k^i)D^i$. Since p_B^i is redundant, we set $p_B^i = 0$ as before. Differentiating with respect to p_S^i and k^i gives the first order conditions

$$D^i + (p_S^i - k^i)\frac{dD^i}{dp_S^i} = 0$$

$$-D^i - (p_S^i - k^i)\frac{dD^i}{dp_S^i}b'(k^i) = 0.$$

In the symmetric equilibrium we have $b'(k^i) = 1$ so that $k^i = k^e$ and $\beta^i = \beta$ for all i, and

$$p_S^N = k^e + \frac{1 - (1 - G(\beta))^N}{N\left((1 - G(\beta))^{N-1}g(\beta) + (N-1)\int_0^\beta (1 - G(c))^{N-2}g(c)^2 dc\right)},$$

where p_S^N is the equilibrium fee N competing intermediaries charge sellers. Given the log-concavity of G, we can show

$$k^e < p_S^N < p_S^m. \tag{4}$$

This establishes that competition reduces the fee below the level set by a monopoly intermediary, although it remains above cost.

With an additional mild technical assumption (for any distribution with $g(0) = 0$, we require $\lim_{c\to 0} \frac{g'(c)}{g(c)}$ to be unbounded), we show that

$$N \to \infty \text{ implies } p_S^N \to k^e. \tag{5}$$

This establishes that when a large number of intermediaries compete, the fee to sellers approximates cost, and the outcome matches that in Section 5 with two intermediaries that are identical price competitors.

With price coherence, buyers join M^i if $b(k^i) - p_B^i - c_i \geq \max\{0, \max_{j\neq i}\{b(k^j) - p_B^j - c_j\}\}$. The demand for M^i is

$$D^i = \int_0^{b(k^i) - p_B^i} \Pi_{j=1,j\neq i}^N (1 - G((b(k^j) - p_B^j) - (b(k^i) - p_B^i) + c_i))dG(c_i)$$

and its profit is $(p_B^i + p_S^i - k^i)D^i$. The equilibrium price of sellers is given by $\hat{p} = d + \sum_{j=1}^N D^j p_S^j + t$. By the same logic as in Section 5, each intermediary wants to extract as much as possible from sellers while keeping them on board, which implies that $p_B^i = 0$ and $p_S^i = b(k^i)$. Thus, the profit of M^i can be written as

$$(b(k^i) - k^i)\int_0^{b(k^i)} \Pi_{j=1,j\neq i}^N (1 - G(b(k^j) - b(k^i) + c_i))dG(c_i).$$

Note that each intermediary attracts more demand by having a higher level of investment than other intermediaries, although with $k > k^e$ this entails a lower margin on existing demand. The equilibrium

level of k^i reflects this tradeoff. Differentiating with respect to k^i gives

$$(b'(k^i) - 1)D^i + (b(k^i) - k^i)\frac{dD^i}{dk^i} = 0. \tag{6}$$

Let k_N denote the solution to (6) in the symmetric equilibrium in which all N intermediaries choose the same investment level. Given the log-concavity of G we can show that

$$k^* < k_N < \overline{k}. \tag{7}$$

This establishes that the N competing intermediaries invest even more in buyer-side benefits than does a monopoly intermediary.

In the limit,

$$N \to \infty \ \text{ implies } \ k_N \to \overline{k}. \tag{8}$$

This establishes that when a large number of intermediaries compete, the investment in buyer-side benefits is so high that the net surplus created by intermediaries is almost fully dissipated, approximating the outcome in Section 5 with two intermediaries that are identical price competitors.

In summary, with multiple differentiated intermediaries, the results lie between the case of a monopoly intermediary and the case of two identical price-setting intermediaries. Given that the harm caused by price coherence is greater in the case of two identical price-setting competing intermediaries compared to the case of a single monopoly intermediary, this shows more generally than Section 5 that competition between intermediaries worsens the distortions caused by price coherence.

6.2 Usage heterogeneity

In this subsection, we explore outcomes when buyer heterogeneity is no longer with respect to the cost of joining an intermediary but rather with respect to the cost of using an intermediary's service. We modify the model of Section 5 by assuming the benefit of *using* an intermediary is $b(k) - c$, where buyers draw c from the distribution G as before. This is equivalent to considering heterogeneity in the benefit buyers obtain.

Since a buyer can only use one intermediary for a given purchase, we assume that each buyer continues to join at most one intermediary. We also adopt the tie-breaking rule that a buyer only joins an intermediary if the buyer plans to use it. These assumptions reflect that there are some (possibly small) costs of joining each intermediary, although we set these to zero in our analysis for simplicity. The assumptions ensure there is still an extensive margin over which our mechanism can apply. Assume that $b(k^e) - k^e > E[c|c < b(k^e)]$, so that the net welfare created by intermediaries at

the efficient investment level is positive even when buyers do not have to pay to use their services. All other assumptions are as in Section 5.

The analysis without price coherence is essentially the same as the case in which c is a joining cost. Fees are passed through to buyers, so intermediaries choose the efficient level of investment k^e. Bertrand-like competition still results in an equilibrium in which $p_S = k^e$. Buyers join and use an intermediary whenever $c < b(k^e) - k^e$, still yielding the first-best outcome.

The analysis with price coherence changes slightly. The main difference results from how much sellers are willing to pay to join an intermediary. A buyer's benefit of using an intermediary for a transaction depends on the buyer's draw of c. A buyer wants to purchase through an intermediary if $c \leq b(k^i) - p^i_B$, in which case the benefit is $b(k^i) - p^i_B - c$. Consistent with merchant internalization, sellers take into account the *expected* net surplus buyers get from purchasing through an intermediary. Thus, sellers are willing to join M^i if $p^i_S \leq b(k^i) - p^i_B - E[c|c < b(k^i) - p^i_B]$. Bertrand-type competition implies that in equilibrium each intermediary M^i offers buyers the maximum possible surplus $b(k^i) - p^i_B$ consistent with cost recovery (i.e. $p^i_B + p^i_S \geq k^i$) and sellers remaining on board $\hat{\beta}^i \equiv b(k^i) - p^i_B - p^i_S - E[c|c < b(k^i) - p^i_B] \geq 0$. Because $\hat{\beta}^i$ is increasing in $b(k^i) - p^i_B$, a property that follows from the log-concavity of G, an intermediary can always lower p^i_B or raise k slightly to attract all the buyers that want to use an intermediary while keeping sellers on board. Intermediaries will do so until the point at which they break-even: $p^i_B + p^i_S = k^i$. Thus, M^i solve $\max_{k^i, p^i_B \geq 0}\{b(k^i) - p^i_B\}$ s.t. $b(k^i) - k^i - E[c|c < b(k^i) - p^i_B] \geq 0$. Given our assumption that $b(k^e) - k^e > E[c|c < b(k^e)]$, M^i does best setting $p^i_B = 0$ and the highest possible k^i so the constraint is binding. Thus, in the symmetric equilibrium, intermediaries set

$$k^i = b(k^i) - E[c|c < b(k^i)]. \tag{9}$$

The equilibrium condition in (9) mirrors that in Section 5 except that c was previously a sunk cost and so the term $E[c|c < b(k^i)]$ did not appear. Here c lowers the net benefit buyers enjoy from buying through an intermediary and hence reduces the fees the intermediaries can extract from sellers. This in turn means that the intermediaries' break-even point for buyer-side investment, denoted k_U, is lower because of usage heterogeneity. There exists a unique level of k_U which solves (9), and it is such that

$$k^e < k_U < \overline{k}. \tag{10}$$

The inequalities imply that there is still over-investment in buyer-side benefits, albeit less so than in

the case without usage heterogeneity.

Both intermediaries choose the same fees and investment levels in equilibrium, and intermediaries make zero profit. With price coherence, buyers join and use an intermediary if $c < b(k_U)$. Since $b(k_U) > b(k^e) > \pi^e$, too many buyers join and use intermediaries. The investment level is also too high $(k_U > k^e)$. Qualitatively, these results mirror those of Section 5.

The price of goods increases by the fee charged to sellers $b(k_U) - E[c|c < b(k_U)]$ which cancels out the net benefits buyers get by purchasing through an intermediary, so the contribution of intermediation to consumer surplus is zero. The net benefit created for buyers ends up exactly equaling the investment cost k_U, so no social benefit is created either. Thus, the positive consumer surplus and welfare obtained without price coherence is fully dissipated with price coherence.

6.3 Membership fees

In this subsection, we explore what happens when intermediaries can set membership fees to buyers. To obtain sharp results, we focus on the case in which intermediaries can observe each buyer's joining cost c and set membership fees accordingly. We start by reconsidering the model of Section 4 with this new assumption.

Without price coherence, M charges a membership fee of $F(c) = b(k) - c$ for a buyer with cost of joining c provided $F(c) \geq k$ so that each buyer generates positive profit. (For higher values of c, M can charge a sufficiently large fee so that the buyer does not join.) With this membership fee, M charges no per-transaction fee; such a fee would have the same effect on buyers' joining decisions as increasing M's membership fee. M therefore obtains a profit of $\max_k\{(b(k) - k - E[c|c < b(k) - k])G(b(k) - k)\}$, and does best maximizing its profit for each buyer, which it does by choosing

$$k = k^e. \tag{11}$$

This implies that without price coherence, M chooses the efficient level of k, consistent with it fully extracting the buyers' surplus through membership fees.

With price coherence, the analysis is the same as without price coherence except that M can recover an additional $b(k)$ from each seller for each unit sold through M. This reflects that the membership fee is a sunk cost, so the usual merchant internalization condition determines how much sellers are willing to pay. As a result, M charges a membership fee of $F(c) = b(k) - c$ to a buyer whose cost of joining is c, provided $F(c) + b(k) \geq k$ so that each buyer generates positive profit. M therefore obtains a profit of $\max_k\{(2b(k) - k - E[c|c < 2b(k) - k])G(2b(k) - k)\}$. Define $k_M = \arg\max_k\{2b(k) - k\}$.

Thus, with price coherence, M chooses k so that

$$k = k_M > k^e. \tag{12}$$

This result reflects that M collects the benefit $b(k)$ twice, once from seller fees and once from membership fees. M overinvests in buyer-side benefits, investing until an extra \$1 of investment generates just \$0.50 in buyer-side benefits.

Too many buyers join M. The number of buyers joining is $G(2b(k_M) - k_M)$. Since $2b(k_M) - k_M \geq 2b(k^*) - k^*$ by the definition of k_M and because $2b(k^*) - k^* > b(k^*)$, even more buyers join than in the case without membership fees (which was $G(b(k^*))$ buyers). Finally, note that M's effect on consumer surplus is also greater compared to the case without membership fees. The effect of higher prices exactly cancels with buyer-side benefits, as in Proposition 3. Buyers' losses instead come from membership fees which extract the surplus that each buyer obtains from joining M net of the buyer's joining cost. As a result, the total consumer surplus loss due to M is $(2b(k_M) - k_M)G(2b(k_M) - k_M)$, which can be large even if joining costs are small. Indeed, because $2b(k_M) - k_M > b(k^*)$ and because $b(k^*) > E[c|c < b(k^*)]$, this loss in consumer surplus is larger than that obtained without membership fees.

Online Appendix Section F considers the case with competing intermediaries. There we show that provided the buyer-side benefit function $b(k)$ is sufficiently flat at the point where the overall net benefits of investment become negative, intermediaries compete membership fees down to zero, meaning that the results of Section 5 are unchanged. This may explain why membership fees are often not observed in practice. Otherwise, membership fees are used, and the results of Proposition 6 in Section 5 become even more pronounced.

6.4   Asymmetric sellers

Throughout, we have focused on equilibria in which all sellers join M. If one or more seller does not join M, buyers who did not join M could go to such sellers and thereby avoid the inflated prices that arise due to price coherence. In Online Appendix Section G, we consider the spokes model of seller competition (Chen and Riordan, 2007), which is a special case of our formulation (1), and consider what happens in the model of Section 4 when one seller (which we call the "alternative" seller) does not join M. As before, each buyer decides whether to join M before learning her "location" (i.e. mismatch cost of each seller), and thus whether she will want to buy from the alternative seller.

While the higher price of buying through M (relative to the alternative seller) influences each

buyer's joining decision, timing dulls the effect in that, at the time when the buyer chooses to join M, there is only a *chance* of wanting to purchase through the alternative seller. M can therefore still benefit by imposing price coherence in equilibrium and setting high fees to sellers such that the remaining sellers are just willing to join M. This continues to reduce consumer surplus. We establish that as the number of sellers becomes large, the results converge to the setting of Section 4 in which all sellers join, so the fact that one seller eschews M makes little difference when there are a large number of sellers that do join. We explore numerically what happens for varying numbers of sellers, including the case where there are only two sellers, so that one joins M in equilibrium and the other does not by assumption. When $G(c)$ takes on the uniform distribution and $b(k)$ is a power function, we confirm that the qualitative results of Proposition 3 continue to hold. When there are fewer sellers, price coherence causes smaller distortions and less loss of consumer surplus, as the alternative seller becomes relatively more important in buyers' decisions.

Asymmetric sellers yield two effects not present in the symmetric setting. First, there is a distortion in the buyers' choice of seller, which does not arise with symmetric sellers. Some buyers that join M end up preferring to buy from a seller that joins M even though they are "located" closer to the alternative seller. Similarly, some buyers that do not join M end up preferring to buy from the alternative seller even though they are "located" closer to another seller. (For example, a buyer might choose to buy at a store that accepts credit cards in order to claim rewards, even though the buyer would have otherwise preferred to buy from another shop that does not accept credit cards.) These distortions add to the loss in consumer surplus caused by price coherence. Second, because merchant internalization no longer holds exactly, all sellers end up slightly worse off in the equilibrium with price coherence compared to the equilibrium that arises without intermediation or the equilibrium without price coherence. This is consistent with widespread seller complaints about price coherence and the fees charged by intermediaries.

7 Policy implications and conclusion

Buyers often view price coherence as a bonus as there is no extra fee to pay by credit card, to book a plane ticket through a travel agent, or to buy insurance through a broker. Indeed, many readers of this paper probably enjoy credit card rebates and other benefits resulting from price coherence. Of course buyers ultimately *do* pay the associated costs: In equilibrium, prices increase to cover sellers' costs of offering "free" benefits. Moreover, including these services in the base price encourages

over-consumption. Specifically, we showed that an intermediary always imposes price coherence if it has the ability to do so, as this increases its profit even though it leads to excessive intermediation and excessive investment in buyer-side benefits, and indeed harms buyers, making them worse off in aggregate compared to the case without any intermediation (Proposition 3). We also showed that these effects persist and even grow when multiple intermediaries compete (Proposition 6). We grounded these theoretical findings in several markets where price coherence is prominent (Section 2).

Competition and consumer protection regulators have been drawn to markets with price coherence. Of the ten markets in Table 1, competition cases and regulatory investigations have occurred or are ongoing in seven (payment cards, travel booking, hotel booking, insurance brokers, insurance comparison services, online marketplaces, and e-books). Regulators' concern is more than coincidental: By offering benefits to buyers at no direct charge, intermediaries cause excessive usage of their services—usage which then lets intermediaries extract significant fees from sellers, indeed beyond even normal monopoly fees.

A natural regulatory response in affected markets is to end whatever regulation, contract provision, or other rigidity enforces price coherence, assuming that it is feasible to do so. Our model confirms the potential benefits of this approach, but we note that favorable outcomes are not guaranteed. For example, regulatory action in Australia in 2003 stopped card networks from imposing price coherence and required them to inform merchants of their right to impose surcharges. Many Australian merchants subsequently added credit card fees, although these were on average double what credit card acquirers charged to merchants, suggesting that some merchants used surcharges to exploit buyers held up at the point of sale. (See Bourguignon *et al.*, 2014, for an analysis of surcharging that allows for this possibility.) In contrast, hold-up might be less likely in other environments, particularly if consumers can compare all-in prices from the outset or if regulation requires that all-in prices be presented from the outset (as in airfare advertising in the US since 2012). Our analysis assumes that in the absence of price coherence, sellers face no other frictions in passing on intermediation costs to buyers. In principle, this neutrality need not hold. For example, menu costs or other frictions could inhibit pass-through in some environments even if intermediaries are not allowed to impose price coherence. In that case, our results on the impact of intermediation may hold even if intermediaries do not impose price coherence rules, and the benefits of banning price coherence rules may be overstated. It is therefore by no means clear that banning price coherence rules always leads to better outcomes.

Alternatively, regulators could directly oversee the fees that intermediaries charge to sellers. Some countries have already implemented this approach in the case of payment card networks by regulat-

ing interchange fees, including European caps on both credit and debit interchange fees (European Commission, 2013) and the U.S. 2010 Durbin Amendment (15 USC §1693o-2) which limit debit card interchange fees. Similarly, in the context of financial service commissions, some regulators simply disallowed certain payments from sellers to intermediaries—requiring that an intermediary's fees be borne solely by the buyers who choose to use the intermediary's services. Rochet and Tirole (2011) consider the regulation of intermediary fees in the context of payment cards, and their approach can be applied to the other markets we study. Where this approach calls for charging buyers directly but such charges are not feasible, a regulator might instead limit fees to sellers by capping fees at efficient costs, thereby eliminating rebates and other excessive benefits.

Relatedly, regulation could seek to separate charges for the underlying product or service, versus charges for the intermediary's service. Recent changes to insurance and financial services sales practices in Australia, Singapore, and the UK typify this approach. In this case, a buyer must compare two separate prices—the base service and the intermediary's fee. Competition among intermediaries may be more likely to reduce the latter, and the resulting market structure may better facilitate direct purchases by a portion of buyers. Indeed, Singapore's regulatory intervention explicitly seeks to facilitate direct purchases by requiring that every insurer offer "basic insurance" directly to consumers at a price that excludes commissions to agents.

Though our model considers only static outcomes, markets with price coherence tend to feature notable dynamics. For one, improved technology often makes direct purchases easier. For example, the Internet lets airlines sell tickets from their own web sites, and some home sellers use the Internet to market their properties—reducing the relative importance of reservation systems and brokers. These advances constrain intermediaries' fees. At the same time, other factors can raise intermediaries' fees. Most notably, we found that competition among intermediaries can spur higher benefits or rebates to buyers and, in turn, higher fees to sellers. This pattern seems to have occurred with payment cards in the U.S., as well as with travel booking networks and several of the other examples we noted.

One might expect an intermediary to impose price coherence only when it achieves significant market power, as it might be difficult to impose price coherence as a new entrant. Yet in practice we observe that in many countries, payment cards imposed price coherence from the outset. Price coherence also began at the outset in other markets including travel reservation services and online restaurant ordering. If each buyer signs up with only a single intermediary, as we assumed, then even a small or new entrant intermediary has market power as to its buyers, when dealing with sellers that want to reach those buyers. This may explain why even small intermediaries succeed in imposing

price coherence. Furthermore, a new intermediary often begins by serving sellers' best customers. For example, credit cards were first used by wealthy consumers and businesses, and business travelers have long used travel agents disproportionately. Edelman and Wright (2015) discusses these and other circumstances in which intermediaries began to impose price coherence.

A limitation of our model is that we do not consider the possibility of buyers using an intermediary to identify or test a suitable product, then purchase through a lower-cost intermediary (or directly). Canonically, a buyer might visit a retail showroom, examine several products, then buy from a seller that avoids costly retail operations (i.e. the problem of "showrooming"). A similar approach could undercut any intermediary that provides a search function. In principle, price coherence can reduce or eliminate the incentive for showrooming by preventing a low-cost intermediary or direct purchase from undercutting a high-cost intermediary. However, an intermediary may be able to limit showrooming in less restrictive ways. For example, an intermediary might try to prevent buyers from switching to direct purchase after obtaining search information.[12] Even if price coherence is necessary to prevent showrooming, price coherence still leads to the same distortions we uncovered. More generally, our theory of harm applies equally no matter the *reason* for price coherence. To determine the net benefit of price coherence, in light of anti-showrooming benefits offset against the distortions we demonstrate, we would need a model in which an intermediary helped buyers search among sellers. We leave this for future work.

Despite the many concerns prompted by price coherence, the intermediaries at issue nonetheless tend to provide significant benefits. One can hardly overstate the impact of airline reservation systems, particularly upon their launch in the 1970's when they constituted the largest private computer network in the world (at the time, eclipsed only by the U.S. Department of Defense). Credit cards, insurance brokers, and other intermediaries can offer similar benefits to the buyers and sellers who most value their services. But price coherence causes the costs of these services to be paid by all, including those who prefer lower-cost alternatives. The resulting market structure, we argue, is ripe for further examination.

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[12]It may be possible to prevent such switching when selling through individualized quotes. Commercial insurance is illustrative: An insurer will typically refuse to quote a single policy (for a single customer) through multiple brokers. Instead, having obtained a quote from a given insurer through a first broker, the buyer can only buy that policy through that broker.

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Appendix: Proofs

This appendix provides proofs of all results.

Proof of Proposition 1

Suppose M sets $k \geq 0$, $p_B \geq 0$ and $p_S \geq 0$ in stage 1. Define $\beta = b(k) - p_B - p_S$. Consider first $\beta > 0$. Consider buyers' and sellers' equilibrium strategies in stage 2. These are: (i) buyers join M if and only if $c \leq \beta$; (ii) all sellers join M; and (iii) each seller i sets the prices $p_i^d = d + t$ and $p_i^m = d + p_S + t$, for direct purchases and purchases through M respectively.

We show that (i)-(iii) characterize an equilibrium in the stage 2 subgame. Given (ii)-(iii), buyers pay $p_B + p_S$ more if they want to buy through M and incur the cost c of joining, but get the extra benefit $b(k)$, so (i) follows. Because a seller can always pass through the higher cost p_S and give its buyers the option of using M, (ii) holds trivially.

To show why (iii) holds, consider first the problem for seller i facing the demand in (1) in the absence of M. Given all other sellers set the same (scalar) price $\hat{p}_{-i}$, seller i chooses p_i to solve:

$$\max_{p_i} \left\{ (p_i - d) \left(\frac{1}{n} + \sigma((v - p_i) - (v - \hat{p}_{-i})) \right) \right\}. \tag{13}$$

The symmetric equilibrium involves $p^* = d + \frac{1}{\sigma n} = d + t$. In equilibrium, each seller obtains the profit of $\pi = \frac{t}{n}$. We refer to the *standard seller problem* as

$$\max_{p_i} \left\{ (p_i - d) \left(\frac{1}{n} + \sigma(d + t - p_i) \right) \right\},$$

and the resulting equilibrium profit as the *standard seller profit*.

Now allow buyers to join M, and suppose seller i sets prices so that $p_i^m - p_i^d \leq b(k) - p_B$. Then buyers who join M purchase through M. Given the other sellers stick to their equilibrium strategies, seller i's profit is

$$\max_{p_i^d, p_i^m} \left\{ (1 - G(\beta))(p_i^d - d) \left(\frac{1}{n} + \sigma(d + t - p_i^d) \right) + G(\beta)(p_i^m - d - p_S) \left(\frac{1}{n} + \sigma(d + t + p_S - p_i^m) \right) \right\}.$$

Note that seller i's profit is separable into two parts: profit from buyers purchasing directly, and profit from those coming through M. The analysis from the standard seller problem applies to both profit terms (in the latter case by setting the seller's cost equal to $d + p_S$). This implies seller i does best

setting $p_i^d = d + t$ and $p_i^m = d + p_S + t$, so as to obtain the standard seller profit. Note that seller i does worse by setting prices so that $p_i^m - p_i^d > b(k) - p_B$, which would make all buyers that choose seller i prefer to buy directly. Seller i's deviation profit in this case is

$$\max_{p_i^d} \left\{ (1 - G(\beta))(p_i^d - d) \left(\frac{1}{n} + \sigma(d + t - p_i^d) \right) + G(\beta)(p_i^d - d) \left(\frac{1}{n} + \sigma(d + t - p_i^d - \beta) \right) \right\}.$$

Comparing this to (13) and recalling that $\beta > 0$, seller i's resulting profit must be strictly less than the standard seller profit. Thus, (iii) holds.

The only other symmetric equilibrium in the continuation game is the trivial equilibrium. Given the equilibrium selection rule adopted, the equilibrium in (i)-(iii) prevails, provided that M can obtain a positive profit, which is confirmed below.

In stage one, M sets $p_B \geq 0$ and $p_S \geq 0$ to maximize $(p_B + p_S - k)G(b(k) - p_B - p_S)$. Without loss of generality, set $p_B = 0$. If $k < k^e$, then by increasing k to k^e and increasing p_S by an equal amount, M's margin on each sale would remain unchanged, but M would sell more units since $b(k)$ would increase by more than p_S given that $b'(k) > 1$ for $k < k^e$. By the same logic, if $k > k^e$, then M can always increase its profit by lowering k to k^e. Thus, M sets $k = k^e$ and p_S to maximize $(p_S - k^e)G(b(k^e) - p_S)$. Since G is differentiable and log-concave, the solution p_S^m satisfies (2). Given that $\pi^e = b(k^e) - k^e > 0$, $k^e < p_S^m < b(k^e)$. This ensures that $\beta > 0$ and that M obtains a positive profit.

Finally, note that M would never set fees such that $\beta \leq 0$ since following the analysis above, buyers would never expect to get any surplus from joining M, and so no buyers would join, leaving M with no profit.

Proof of Proposition 2

Suppose M sets $0 \leq k \leq \bar{k}$, $p_B \geq 0$ and $p_S \geq 0$ in stage 1.[13] Consider buyers' and sellers' equilibrium strategies in stage 2. These are: (i) buyers join M if and only if $c \leq b(k) - p_B$ and $\beta \geq 0$; (ii) if $\beta \geq 0$ then all sellers join M and set the price $\hat{p} = d + G(b(k) - p_B)p_S + t$; (iii) if $\beta < 0$ then no sellers join M and sellers price at $d + t$.

We show that (i)-(iii) characterize an equilibrium in the stage 2 subgame. Given (ii)-(iii), buyers pay p_B more if they want to buy through M, but get the extra benefit $b(k)$, so are willing to join whenever sellers are expected to join (i.e. $\beta \geq 0$) and $c \leq b(k) - p_B$. This establishes (i).

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[13] Note $k \leq \bar{k}$ holds since at any higher k, even if M can extract the full benefit buyers obtain from going through M in its fees and even if all buyers join, M would make a loss.

Suppose $\beta \geq 0$. If sellers join and set the proposed equilibrium price, they obtain the standard seller profit. Now suppose seller i deviates, does not join M, and sets the price p_i'. Its deviation profit is

$$\max_{p_i'} \left\{ \begin{array}{c} G(b(k) - p_B)(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i' - (b(k) - p_B)))\right) \\ +(1 - G(b(k) - p_B))(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i')\right) \end{array} \right\}, \tag{14}$$

which after substituting in $\hat{p}$ and rearranging terms, can be written as[14]

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma(d + t - p_i' - G(b(k) - p_B)\beta)\right) \right\}. \tag{15}$$

Compare the profit in (15) with the profit in (13). The profit expression is the same, with the same margin, with demand the same if $\beta = 0$ and lower if $\beta > 0$. Since the profit in (13) is the same as the profit in the proposed equilibrium (i.e. standard seller profit), each seller i is willing to join M given that $\beta \geq 0$ and that buyers and the other sellers follow their equilibrium strategies. This establishes (ii). Given the equilibrium selection rule adopted, the equilibrium in (i)-(ii) prevails if $\beta \geq 0$ provided that M obtains a positive profit, which is confirmed below.

In case $\beta < 0$, the above analysis implies that the deviation profit of seller i is higher than the standard seller profit, so there is no equilibrium in which all sellers join M. Instead, the only symmetric equilibrium is the trivial equilibrium. The standard seller problem applies, and sellers price at $d + t$ in equilibrium. This establishes (iii).

In stage one, M sets $p_B \geq 0$, $p_S \geq 0$ and k to maximize the resulting profit $(p_B + p_S - k)G(b(k) - p_B)$ subject to $p_S \leq b(k) - p_B$. This is maximized by setting $p_B = 0$, $p_S = b(k)$ and $k = k^*$ as defined in (3). Since $G(b(k))$ is increasing in k at k^e, $k^* > k^e$. Moreover, since $b(k) - k$ is decreasing for $k > k^e$ and reaches zero when $k = \overline{k}$, it must be that $k^* < \overline{k}$ so that M obtains a positive profit. Given that G is log-concave in its argument and $b(k)$ is strictly concave in k, the function $(b(k) - k)G(b(k))$ is log-concave in k, implying there exists a unique k^* satisfying these properties. The remaining properties of the equilibrium outcome stated in the proposition follow by substituting M's optimal choices into (i)-(iii).

Proof of Proposition 4

Suppose each M^i ($i = 1, 2$) sets $k^i \geq 0$, $p_B^i \geq 0$ and $p_S^i \geq 0$ in stage 1. Suppose first $\beta^i > 0$ for $i = 1, 2$. Consider buyers' and sellers' equilibrium strategies in stage 2. These are: (i) buyers join M^1 if $c \leq \beta^1$

---

[14]The assumptions that $t > b(\overline{k})$ and $\beta \geq 0$ ensure that each of the market shares in (14) lies between 0 and 1 at seller i's optimal deviation price.

and $\beta^1 > \beta^2$; buyers join M^2 if $c \leq \beta^2$ and $\beta^2 > \beta^1$; buyers randomize between M^1 and M^2 if $c \leq \beta^1$ and $\beta^1 = \beta^2$ (otherwise, they do not join either intermediary); (ii) all sellers join M^1 and M^2; and (iii) each seller sets the price $p^d = d + t$ for buyers that purchase directly and the prices $p^j = d + p_S^j + t$ for buyers that purchase through M^j.

We show that (i)-(iii) characterize an equilibrium in the stage 2 subgame. Given (ii)-(iii), buyers pay $p_B^i + p_S^i$ more if they want to buy through M^i and incur the cost c of joining, but get the extra benefit $b(k^i)$, so (i) follows. (Note joining both intermediaries would require a buyer to incur the cost c twice with no extra benefit, so can be ruled out).

Sellers can never do better by not joining one of the intermediaries, say M^i. A seller can always pass through the higher cost p_S^i so as to maintain the same margin as if buyers purchase directly, but given $\beta^i > 0$ the seller attracts at least as many buyers by joining M^i since the seller can offer a greater surplus to any buyers that have joined M^i. Thus, (ii) holds.

To show why (iii) holds, first note that if $\beta^j < \beta^i$ then no buyers join M^j and so the sellers cannot do better than set the prices p^j given in (iii). If $\beta^j > \beta^i$, then buyers join M^j if they join at all, and the proof of (iii) follows exactly as in the proof of Proposition 1 with p_i^m replaced by p_i^j, p_S replaced by p_S^j, and β replaced by β^j. In case $\beta^j = \beta^i$, sellers expect an equal number of buyers to join each intermediary, and given the other sellers stick to their equilibrium strategies, seller i's profit is separable into the profit from buyers purchasing directly, from buyers purchasing through M^1 and from buyers purchasing through M^2. The same analysis applies to each profit expression following the proof of Proposition 1, which establishes (iii).

In case $\beta^i \leq 0$ for any i, the above analysis implies that no buyers would join M^i, so M^i would make no profit and would not choose such fees.

The stage one analysis then follows directly, as explained in the paragraph preceding the statement of Proposition 4. The intermediaries make zero profit in equilibrium. The remaining properties of the equilibrium outcome stated in the proposition follow by substituting the intermediaries' equilibrium choices in stage one into (i)-(iii).

Proof of Proposition 5

Extending the approach taken in the proof of Proposition 2, suppose each M^i ($i = 1, 2$) sets $0 \leq k^i \leq \bar{k}$, $p_B^i \geq 0$ and $p_S^i \geq 0$ in stage 1. Consider buyers' and sellers' equilibrium strategies in stage 2. These are: (i) if $\beta^i \geq 0$ and $b(k^i) - p_B^i > b(k^j) - p_B^j$, then buyers only join M^i (doing so if they draw

$c \leq b(k^i) - p_B^i$) and sellers join both intermediaries and set the price $\hat{p}^i = d + t + G(b(k^i) - p_B^i)p_S^i$. (ii) if $\beta^i \geq 0$, $\beta^j < 0$ and $b(k^i) - p_B^i \leq b(k^j) - p_B^j$, then the same strategies hold as in (i) except sellers only join M^i. (iii) if $\beta^1 \geq 0$, $\beta^2 \geq 0$ and $b(k^1) - p_B^1 = b(k^2) - p_B^2$, which can be denoted $b(k) - p_B$, then buyers join an intermediary if they draw $c \leq b(k) - p_B$, randomizing over which intermediary to join, while all sellers join both intermediaries and set the price $\hat{p} = d + G(b(k) - p_B)\left(\frac{p_S^1}{2} + \frac{p_S^2}{2}\right) + t$. (iv) if $\beta^1 < 0$ and $\beta^2 < 0$, then no buyers or sellers join either intermediary, and each seller prices at $d + t$.

We show that (i)-(iv) characterize an equilibrium in the stage 2 subgame. Consider case (i). Buyers strictly prefer to join M^i over M^j, and do so whenever they draw $c \leq b(k^i) - p_B^i$. Given buyers only join M^i, sellers' optimal strategies are determined exactly as in case (ii) in the proof of Proposition 2 in which there is a single intermediary, so they are willing to join M^i if and only if $\beta^i \geq 0$, and they set the price $\hat{p}^i = d + t + G(b(k^i) - p_B^i)p_S^i$. The only difference is that sellers also join M^j, since they cannot do better not joining M^j. This establishes (i).

Consider case (ii). If sellers join both M^i and M^j, then buyers prefer to join M^j exclusively. The proof of Proposition 2 then implies that sellers would not want to join M^j. Rather, it is an equilibrium for sellers to join M^i only following the same analysis as in (i). This establishes (ii).

Consider case (iii). Given that sellers join both intermediaries, buyers that join an intermediary are indifferent between joining M^1 and M^2 since they face the same benefits from going through each intermediary and the same seller price due to price coherence. They therefore cannot do better than to randomize between the two intermediaries, so each intermediary attracts half the buyers that join an intermediary. Given that the other sellers join both intermediaries and price at $\hat{p}$, if seller i does likewise, it obtains the standard seller profit. If it deviates and does not join either intermediary and sets its price to be p_i', its deviation profit is equivalent to the expression (14). Substituting in $\hat{p}$ from (iii), its deviation profit can be written as

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma\left(d + t - p_i' - G(b(k) - p_B)\left(\frac{\beta^1 + \beta^2}{2}\right)\right)\right) \right\}. \tag{16}$$

Compare the profit in (16) with the profit in (13). The profit expression is the same, with the same margin and demand at every price if $\beta^1 + \beta^2 = 0$, but lower demand if $\beta^1 + \beta^2 > 0$. Thus, since the profit in (13) is the same as the profit in the proposed equilibrium (i.e. standard seller profit), each seller i does not want to deviate in this way, given that $\beta^1 \geq 0$ and $\beta^2 \geq 0$ and that buyers and the other sellers follow their equilibrium strategies.

The alternative deviation is that seller i only joins one intermediary, say M^j, and sets its price to

be p'_i for buyers purchasing through M^j or directly. Then seller i's deviation profit is

$$
\max_{p'_i}
\left\{
\begin{array}{c}
\frac{1}{2}G(b(k) - p_B)(p'_i - d)\left(\frac{1}{n} + \sigma(\hat{p} - p'_i - (b(k) - p_B))\right) \\[4pt]
+ \frac{1}{2}G(b(k) - p_B)(p'_i - d - p^j_S)\left(\frac{1}{n} + \sigma(\hat{p} - p'_i)\right) \\[4pt]
+ (1 - G(b(k) - p_B))(p'_i - d)\left(\frac{1}{n} + \sigma(\hat{p} - p'_i)\right)
\end{array}
\right\}.
\tag{17}
$$

Substituting the expression for $\hat{p}$ from (iii) above into (17), and solving for the optimal p'_i, seller i still sells to some of the buyers who join M^i (i.e. the expression in the first line of (17) is positive) given our assumption that $t = \frac{1}{\sigma n} > b(\bar{k})$. The difference between the resulting deviation profit and the standard seller profit is proportional to

$$
(\beta^i)^2 G(b(k) - p_B) - 8t\beta^i - 4(b(k) - p_B)p^j_S G(b(k) - p_B).
\tag{18}
$$

Given $t > b(\bar{k}) > \beta^i \geq 0$ and given $b(k) - p_B \geq 0$ since $\beta^j \geq 0$, the expression in (18) is strictly negative. Thus, seller i is strictly worse off deviating in this way, which establishes (iii).

Consider case (iv). The same analysis as above implies that there is no equilibrium in which sellers join both intermediaries. Instead, the trivial equilibrium is selected; buyers and sellers do not join either intermediary. The analysis of the standard seller problem applies, and sellers price at $d + t$ in equilibrium. This establishes (iv).

The stage one analysis then follows directly, as explained in the paragraph proceeding Proposition 5. The intermediaries make zero profit in equilibrium. The remaining properties of the equilibrium outcome stated in the proposition follow by substituting the intermediaries' equilibrium choices in stage one into (i)-(iv).

Price Coherence and Excessive Intermediation: Online Appendix

This online appendix supplements the main text by providing two additional results that are referenced in the main text, along with the full proofs of results from Section 6.

A   Fixed investment costs

Consider our analysis in Section 4 in the main text. In this section, we show that Proposition 3 continues to hold even if the investment cost k is a fixed cost rather than a cost per transaction. This implies that M faces no cost of providing additional buyers with the transactional benefits of intermediation. The only change from Proposition 3 is that M under-invests in k in the absence of price coherence, rather than investing efficiently. This reflects that fewer buyers join and use M in the absence of price coherence compared to the first-best level, which reduces the payoff to the fixed investment.

Since k is set in stage 1, the equilibrium analysis in stage 2 is the same as in Section 4 in the main text where the investment is a cost per transaction. Consider M's choice in stage 1. Without price coherence, M sets $p_B = 0$ and chooses k and p_S to maximize $p_S G(b(k) - p_S) - k$. The optimal p_S^m satisfies $p_S^m = \frac{G(b(k^m) - p_S^m)}{g(b(k^m) - p_S^m)}$, where k^m is the optimal investment level that solves

$$b'(k^m)G(b(k^m) - p_S^m) - 1 = 0.$$

With price coherence, M sets $p_B = 0$ and $p_S = b(k)$, and chooses k to maximize $b(k)G(b(k)) - k$. The optimal k^f satisfies

$$b'(k^f)G(b(k^f)) + b(k^f)g(b(k^f))b'(k^f) - 1 = 0.$$

Clearly, M prefers price coherence, given that $\max_{k,p_S \geq 0}\{p_S G(b(k) - p_S) - k\}$ is always smaller than $\max_k\{b(k)G(b(k)) - k\}$. The derivative of M's profit function at k^m is

$$b'(k^m)G(b(k^m)) + b(k^m)g(b(k^m))b'(k^m) - 1 > b'(k^m)G(b(k^m) - p_S^m) - 1 + b(k^m)g(b(k^m))b'(k^m)$$

$$> 0.$$

Thus, M optimally chooses k^f above k^m.

Total welfare

$$W = \int_0^{b(k)} (b(k) - c)dG(c) - k = b(k)G(b(k)) - k - \int_0^{b(k)} cdG(c),$$

is maximized at the k^w that solves

$$b'(k^w)G(b(k^w)) - 1 = 0.$$

Comparing the first-order conditions, clearly $k^m < k^w < k^f$ given that $b'(k) > 0$. Thus, M under-invests without price coherence, and M over-invests with price coherence. Since buyers join when $c < b(k^m) - p_S^m$ without price coherence and when $c < b(k^f)$ with price coherence, too few buyers join without price coherence and too many buyers join with price coherence. The effect on consumer surplus is also the same as Proposition 3, following the same logic. Consumer surplus generated by M is positive without price coherence and negative with price coherence.

B Condition for buyers to always purchase

Consider our analysis in Section 4 in the main text. Throughout, we assumed that buyers always purchase from one of the sellers. In this section, we provide a condition on v to ensure that buyers always purchase in the equilibria we have characterized. Let τ be the maximum mismatch a buyer can incur going to any seller. In the standard Hotelling model, $\tau = t$. The condition

$$v > d + t + \tau + b(\overline{k}) \tag{19}$$

ensures that buyers always purchase. To see why this condition suffices, consider the least favorable case in the proof of Proposition 1: A buyer has the maximal mismatch with the seller in question and does not purchase through M, so pays $d + t$. In this case, the buyer's surplus is at least $v - d - t - \tau$, which is positive. Likewise, consider the least favorable case in the proof of Proposition 2: A buyer has the maximal mismatch with the seller in question and does not purchase through M, and so pays the maximum possible price $d + t + b(\overline{k})$ which would arise if $p_S = b(k)$ and $p_B = 0$ and k is set to the highest possible level at which M still breaks even. In that case, the buyer's surplus is $v - d - t - \tau - b(\overline{k})$, which is positive.

The reasoning for the remaining cases in the main text is the same. The only exception is if the equilibrium level of investment k exceeds $\overline{k}$, which can arise when intermediaries can set membership fees (i.e. Section 6.3). Then $\overline{k}$ in (19) is replaced by k_M defined in Section 6.3 in case $k_M > \overline{k}$.

C Elastic demand with costless rebates

We modify our benchmark model in Section 3 in the main text by assuming that k is fixed with the corresponding buyer-side benefits fixed at b (i.e. $b = b(k)$). Assume $0 \leq k < b$. Instead, we allow M

to provide buyer-side benefits by setting $p_B < 0$ so M can offer "costless rebates," incurring only a cost equal to the amount of the rebate.

We generalize the specific formulation of seller competition in (1). We allow that there are $n \geq 1$ symmetric sellers. (Thus, we allow for the case of a monopoly seller). In the absence of M, assume that the sellers' pricing problem is well behaved. Specifically, if buyers face a per-unit price of p_i from each seller i, suppose that the resulting demand function $q_i(p_1, ..., p_n)$ is symmetric with respect to all prices other than p_i, decreasing in p_i, non-decreasing in p_j ($j \neq i$), and twice differentiable on the interval of prices over which it is non-negative. If all sellers set the same price p, let $q(p)$ denote the common demand faced by all sellers. Denote the corresponding indirect utility $v(p)$, where $v(p)$ is continuous and decreasing in p with $v(p) = 0$ for some sufficiently large p. Assume that there is a unique symmetric equilibrium price $p^*(x)$ which solves $\arg\max_{p_i}\{(p_i - x)q_i(p_1, ..., p_n)\}$ for $i = 1, ..., n$ in case all sellers have constant unit cost x, and which is characterized by the usual first-order conditions. Also assume that the pass-through of a common cost shock into the symmetric equilibrium price is positive but no more than one (i.e. $0 < dp^*(x)/dx \leq 1$), provided that demand remains positive.[15] Each seller faces a cost of d per unit sold. Assume that $q(p^*(d)) > 0$, so in the absence of M, demand is positive at the equilibrium price. These properties are consistent with a standard discrete choice model in which each buyer chooses at most one seller to buy from, and with representative consumer models in which each buyer chooses to buy some units from all sellers.

We assume that b is not too high. Specifically, we make the mild assumption that the sellers' equilibrium markup (in the absence of M) is at least equal to the net per-unit benefit produced by the intermediary (i.e. $p^*(d) - d \geq b - k$). We replace our previous assumption on $\bar{c}$ with the assumption that $v(p^*(d + k - b)) - v(p^*(d)) < \bar{c} < v(p^*(d))$. This ensures that (i) if M's fees are regulated at cost (i.e. k) and passed onto buyers, not all buyers join M and (ii) the surplus a buyer obtains from the good at the equilibrium price in the absence of M is always higher than the buyer's cost of joining M.

With these changes to our benchmark model, we first consider what happens without price coherence, then what happens when price coherence is imposed.

C.1 Intermediation without price coherence

Without price coherence, the results are similar to our benchmark case. It makes no difference whether M charges buyers directly or instead charges sellers who can reflect these charges in their own prices to buyers. Because sellers can set differential prices, they set their symmetric equilibrium price $p^*(d)$

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[15]This is implied by log-concavity of $q(p)$ with respect to p if there is only one seller.

for direct purchases. Each seller i joins M and sets its price for intermediated purchases to solve $\max_{p_i}\{(p_i - d - p_S)q_i(p_1 + p_B - b, ..., p_n + p_B - b)\}$ assuming that buyers that have joined M prefer to purchase through M for any seller than purchasing directly. (We will confirm that this is true in equilibrium.) With a change of variables ($p_i' = p_i + p_B - b$), this is equivalent to the problem $\max_{p_i'}\{(p_i' - d + b - p_B - p_S)q_i(p_1', ..., p_n')\}$. The symmetric equilibrium implies that the common price for intermediated purchases is $p' = p^*(d + p_B + p_S - b)$, and so $p' + b - p_B$ in the original problem. The effective price, after taking into account the net benefit $b - p_B$ per unit, is therefore $p^*(d + p_B + p_S - b)$, and the marginal buyer is determined by $\tilde{c} = v(p^*(d + p_B + p_S - b)) - v(p^*(d))$. To attract any buyers, it must be that $p_B + p_S < b$. This ensures that the effective price facing buyers is lower when purchasing through M and thereby ensures that all buyers that join M will indeed prefer to purchase through M.

M's profit is therefore $G(\tilde{c})(p_B + p_S - k)q(p^*(d + p_B + p_S - b))n$, subject to $p_B + p_S \leq b$. Since M's profit only depends on the sum $p_B + p_S$, we set $p_B = 0$ without loss of generality. The optimal p_S satisfies $k < p_S < b$ so that M makes a positive profit. Given that the pass-through of a common cost shock is no more than one, $p^*(d + p_S - b) + b > p^*(d)$ for $0 < p_S < b$, so the price sellers set for intermediated purchases is higher than the price they set for direct purchases. Given $v(p^*(d + k - b)) - v(p^*(d)) < \overline{c}$ and since $p_S > k$, it must be that $\tilde{c} = v(p^*(d + p_S - b)) - v(p^*(d)) < \overline{c}$. (Here $\tilde{c}$ is the threshold level of c below which buyers join M and above which buyers do not join M.) Thus, some but not all buyers join M.

We can summarize these key results when M cannot impose price coherence: There exists an equilibrium in which M sets the fees $p_B = 0$ and $k < p_S < b$. All sellers join M and set the equilibrium price $p^*(d)$ for buyers that purchase directly and the price $p^*(d + p_S - b) + b$ for buyers that purchase through M, with $p^*(d + p_S - b) + b > p^*(d)$. Some but not all buyers join M (i.e. $0 < \tilde{c} < \overline{c}$).

As before, this equilibrium is equivalent to other symmetric equilibria with different p_B and p_S but the same total fee $p_B + p_S$, in that all such equilibria yield identical decisions by buyers and sellers, as well as identical amounts paid or received by each agent.

C.2   Impact of price coherence

Suppose M imposes price coherence. Define $\tilde{p}_B$ such that $v(p^*(d)) - v(p^*(d) + b - \tilde{p}_B) = \overline{c}$. We will show that if $p_S = b - \tilde{p}_B$ then $\tilde{p}_B$ exists such that all buyers and sellers join M.

If all buyers join M, a seller gets profit $\max_{p_i}\{(p_i - d - p_S I_i)q_i(p_1 + (\tilde{p}_B - b)I_1, ..., p_n + (\tilde{p}_B - b)I_n)\}$, where I_i is an indicator variable which equals zero if seller i does not join M and equals one if seller

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i joins M. With a change of variables $(p'_i = p_i + (\tilde{p}_B - b)I_i)$, the sellers' problem can be rewritten equivalently as $\max_{p'_i}\{(p'_i - d - (\tilde{p}_B + p_S - b)I_i)q_i(p'_1, ..., p'_n)\}$ for $i = 1, ..., n$. Seller i's effective marginal cost in this rewritten problem is $d + (\tilde{p}_B + p_S - b)I_i$, so a seller joins M if and only if $\tilde{p}_B + p_S \leq b$. Provided that this constraint holds, the symmetric equilibrium implies the common price $p' = p^*(d + \tilde{p}_B + p_S - b)$, and so $p' + b - \tilde{p}_B$ in the original problem. The effective price, after taking into account the net benefit $b - \tilde{p}_B$ per unit, is therefore $p^*(d + \tilde{p}_B + p_S - b)$, and the equilibrium quantity is $q(p^*(d + \tilde{p}_B + p_S - b))$.

With $p_S = b - \tilde{p}_B$, each seller is just willing to join, and sets the common price $p^*(d) + b - \tilde{p}_B$ in equilibrium. If a buyer purchases directly, she gets the indirect utility $v(p^*(d) + b - \tilde{p}_B)$. If the buyer joins M, she gets $v(p^*(d))$ but incurs the cost c. Given the definition of $\tilde{p}_B$, all buyers join M. The existence of $\tilde{p}_B$ follows since (i) if $p_B = b$, we have $v(p^*(d)) - v(p^*(d) + b - p_B) = 0$; (ii) if p_B is sufficiently negative, then $v(p^*(d) + b - p_B) = 0$ so $v(p^*(d)) - v(p^*(d) + b - p_B) > \bar{c}$; and (iii) because $v(p^*(d) + b - p_B)$ is continuous and increasing in p_B, the intermediate value theorem implies $\tilde{p}_B$ exists and is uniquely defined.

If it attracts all buyers, M solves the following problem

$$\max_{p_S \leq b - \tilde{p}_B} \{(\tilde{p}_B + p_S - k)q(p^*(d + \tilde{p}_B + p_S - b))\}. \tag{20}$$

With a change of variables, this is equivalent to

$$\max_{p'_S \leq b}\{\Pi(p'_S)\}, \tag{21}$$

where $\Pi(p'_S) = (p'_S - k)q(p^*(d + p'_S - b))$. Differentiating $\Pi(p'_S)$ with respect to p'_S and evaluating for $k \leq p'_S < b$, we have

$$\frac{d\Pi(p'_S)}{dp'_S} = q(p^*(d + p'_S - b)) + (p'_S - k)\frac{dq(p^*(d + p'_S - b))}{dp^*(d + p'_S - b)}\frac{dp^*(d + p'_S - b)}{dp'_S} \tag{22}$$

$$\geq q(p^*(d + p'_S - b)) + (p'_S - k)\frac{\partial q_i(p^*(d + p'_S - b))}{\partial p_i}\frac{dp^*(d + p'_S - b)}{dp'_S} \tag{23}$$

$$= q(p^*(d + p'_S - b))\left(1 - \frac{p'_S - k}{p^*(d + p'_S - b) - (d + p'_S - b)}\frac{dp^*(d + p'_S - b)}{dp'_S}\right) \tag{24}$$

$$\geq q(p^*(d + p'_S - b))\left(1 - \frac{p'_S - k}{p^*(d) - d}\right) \tag{25}$$

$$> q(p^*(d + p'_S - b))\left(1 - \frac{b - k}{p^*(d) - d}\right) \tag{26}$$

$$\geq 0, \tag{27}$$

where (22) follows from differentiating $\Pi(p'_S)$ with respect to p'_S, (23) follows from the fact that $\partial q_i/\partial p_j \geq 0$ for $j \neq i$, (24) follows from the first-order condition for each seller's profit-maximizing

choice of price evaluated at the symmetric equilibrium, (25) follows from the pass-through of a common cost shock being no more than one, (26) follows since $p'_S < b$, and (27) follows from our assumption that $p^*(d) - d \geq b - k$. Thus, M solves (21) by making the constraint binding (i.e. setting $p'_S = b$), which implies that M solves (20) by setting $p_S = b - \tilde{p}_B$.

Thus, provided that M does best by attracting all buyers, the key implications of price coherence are: M sets fees $p_B = \tilde{p}_B$ and $p_S = b - \tilde{p}_B$, where $v(p^*(d)) - v(p^*(d) + b - \tilde{p}_B) = \bar{c}$. All sellers join M and set a price $p^*(d) + b - \tilde{p}_B$.

Provided that all buyers join, each seller is willing to pay up to $b - \tilde{p}_B$ per unit for transactions facilitated by M since this allows the seller to increase its price by an equal amount without losing any demand. Thus, merchant internalization continues to hold. Compared to the case without price coherence, M sets higher fees (b in total, instead of $p_S < b$) but attracts all buyers instead of only some, so is strictly better off by imposing price coherence. Indeed, compared to the model in Section 3, there is one additional channel through which price coherence helps M. By making the price the same for intermediated and non-intermediated transactions, M lowers the effective price for intermediated transactions and raises it for non-intermediated transactions, thereby increasing the demand from each buyer for intermediated transactions. In the model in Section 3 in the main text, where each buyer has unit demand, this channel does not operate.

Without price coherence, too few buyers join. This reflects that M has market power. It sets $p_B + p_S > k$ whereas the efficient fees are such that $p_B + p_S = k$ given that M has to cover its costs. With price coherence, all buyers join and use M, but under our assumptions not all buyers would join if M just set its fees to cover its costs, so there is excessive intermediation.

Consider what happens to consumer surplus. Without price coherence, each buyer that purchases through M faces an increased price, but prices increase by some amount less than b, so these buyers are better off. Moreover, buyers that do not join M see no change in surplus because they continue to purchase at the monopoly price $p^*(d)$. Thus, without price coherence, intermediation increases consumer surplus. With price coherence, prices exceed the equilibrium price $p^*(d)$ by exactly $b - \tilde{p}_B$. Thus, the benefit buyers get from using M exactly cancels out the higher price they pay for goods, and the net effect of M on buyers is the additional cost of joining, $E[c] = \int_0^{\bar{c}} c \, dG(c)$.

These findings imply the main results in Proposition 3 extend to the case with a general model of seller competition with elastic aggregate demand in which M can use costless rebates. We have shown that provided all buyers join in the equilibrium with price coherence: M always imposes price coherence if permitted to do so; too few buyers join M without price coherence; too many buyers join

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M with price coherence; and consumer surplus increases when M operates without price coherence but decreases when M operates with price coherence, compared to the case in which M does not operate at all.

C.3 Linear demand examples

The results in Section C.2 rely on all buyers joining M in equilibrium. That is, they assume that M cannot do better by raising p_B so that not all buyers join. Without all buyers joining, there are two possibilities for profitable deviations by M. The first possibility is that each individual seller would want to join even if $p_S > b - p_B$ since merchant internalization no longer always holds exactly. This would allow M to increase its margin $p_B + p_S - k$ above $b - k$, possibly raising its profit despite fewer buyers joining and buyers purchasing less. The second possibility is that M does better by lowering its margin $p_B + p_S - k$ below $b - k$ so as to lower the prices set by sellers and thereby expand demand for intermediated transactions. This could increase M's profit if the increase in demand for intermediated transactions outweighs M's lower margin. The analysis in Section C.2 shows this is not possible when all buyers join M, but we have not ruled this out in the case that only some buyers join M.

Both possibilities can be ruled out in the model of seller competition with linear demand defined by (1). The first possibility is ruled out because sellers will only join in that model if $p_S \leq b - p_B$, even if only some buyers join. The second possibility is ruled out because buyers only buy one unit in that model, so demand per buyer is fixed. Note that the other assumptions on demand that we required above also hold for (1).

Both possibilities can also be ruled out for a monopoly seller facing linear demand, or more generally, for competing differentiated sellers facing linear demand, both of which exhibit elastic aggregate demand. We establish this by adopting the demand specification of Shubik and Levitan (1980). Consider each buyer having the utility function

$$U = \mu \sum_{i=1}^{n} q_i - \frac{n}{2} \left((1 - \gamma) \sum_{i=1}^{n} q_i^2 + \frac{\gamma}{n} \left(\sum_{i=1}^{n} q_i \right)^2 \right) + y,$$

with $0 \leq \gamma < 1$ being a measure of product differentiation, and $\mu > d$ where d is the marginal cost of sellers. The extra term y is expenditure on a composite good. Given the budget constraint $\sum_{i=1}^{n} p_i q_i + y \leq Y$, where Y is income, first order conditions for buyers' constrained utility maximization problem are

$$p_i = \mu - n \left((1 - \gamma) q_i + \frac{\gamma}{n} \sum_{i=1}^{n} q_i \right).$$

Therefore

$$q_i = \frac{1}{n}\left(\mu - \frac{p_i}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{\sum_{i=1}^{n}p_i}{n}\right). \tag{28}$$

Note that aggregate demand with n sellers is $\mu - \frac{\sum_{i=1}^{n}p_i}{n}$, which does not depend on the differentiation parameter. If $n = 1$, this captures the case of a monopoly seller facing linear demand.

Each seller solves

$$\max_{p_i}(p_i - d)\left(\frac{1}{n}\left(\mu - \frac{p_i}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{\sum_{i=1}^{n}p_i}{n}\right)\right),$$

implying the symmetric equilibrium price

$$p^*(d) = \frac{n(1-\gamma)\mu + (n-\gamma)d}{(2-\gamma)n - \gamma}.$$

Thus, $p^*(d) - d \geq b - k$ implies $\frac{n(1-\gamma)(\mu-d)}{(2-\gamma)n-\gamma} \geq b - k$. Define $\bar{p}_B = b - \frac{n-\gamma}{(2-\gamma)n-\gamma}(\mu - d)$ which solves $v(p^*(d) + b - \bar{p}_B) = 0$. By the definition of $\tilde{p}_B$, $\tilde{p}_B > \bar{p}_B$.

Suppose M sets p_B high enough ($p_B > \tilde{p}_B$) such that $0 < G < 1$ buyers join. First we show that if M wants to attract all sellers, it must set $p_S < b - p_B$. This will rule out the first possible deviation by M.

If all sellers join, seller i's profit is

$$\max_{p_i}\left\{ \begin{array}{l} G(p_i - d - p_S)\left(\frac{1}{n}\left(\mu - \frac{p_i+p_B-b}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{\sum_{i=1}^{n}(p_i+p_B-b)}{n}\right)\right) \\ +(1-G)(p_i - d)\left(\frac{1}{n}\left(\mu - \frac{p_i}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{\sum_{i=1}^{n}p_i}{n}\right)\right) \end{array} \right\}.$$

Thus the symmetric equilibrium price is

$$p^* = \frac{n(1-\gamma)\mu + (n-\gamma)d + (n-\gamma)Gp_S + n(1-\gamma)G(b-p_B)}{(2-\gamma)n - \gamma},$$

and the equilibrium profit is

$$\pi^* = \frac{1}{n}G(p^* - d - p_S)(\mu - p^* - p_B + b) + \frac{1}{n}(1-G)(p^* - d)(\mu - p^*).$$

For this to be an equilibrium, we require

$$p^* > d + p_S \tag{29}$$

and

$$\mu > p^*. \tag{30}$$

If seller i deviates by not joining, its deviation profit is

$$\max_{p_i'}\left\{ \begin{array}{l} G(p_i' - d)\left(\frac{1}{n}\left(\mu - \frac{p_i'}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{p_i'+(n-1)(p^*+p_B-b)}{n}\right)\right) \\ +(1-G)(p_i - d)\left(\frac{1}{n}\left(\mu - \frac{p_i'}{1-\gamma} + \frac{\gamma}{1-\gamma}\frac{p_i'+(n-1)p^*}{n}\right)\right) \end{array} \right\}.$$

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Its optimal deviation price is

$$p' = \frac{2(n(1-\gamma)\mu + (n-\gamma)d) + (n-1)\gamma G(p_S + p_B - b)}{2((2-\gamma)n - \gamma)},$$

and its resulting deviation profit is

$$
\begin{aligned}
\pi' = & \frac{1}{n} G(p' - d) \left(\mu - \frac{n-\gamma}{n(1-\gamma)} p' + \frac{(n-1)\gamma}{n(1-\gamma)} (p^* + p_B - b) \right) \\
& + \frac{1}{n} (1 - G)(p' - d) \left(\mu - \frac{n-\gamma}{n(1-\gamma)} p' + \frac{(n-1)\gamma}{n(1-\gamma)} p^* \right).
\end{aligned}
$$

Suppose $p_S = b - p_B + \delta$. Then

$$
\begin{aligned}
\pi^* - \pi' = & - \frac{G(1-G)p_S^2}{n} \\
& - \frac{4Gn(1-\gamma)((n-\gamma)(\mu - d) - (1-G)((2-\gamma)n - \gamma)p_S)\delta + G^2((3\gamma - 2)n - \gamma)(n-\gamma)\delta^2}{4n^2(1-\gamma)((2-\gamma)n - \gamma)}.
\end{aligned}
$$

$\pi^* - \pi'$ is negative for any $\delta \geq 0$ if

$$4Gn(1-\gamma)((n-\gamma)(\mu - d) - (1-G)((2-\gamma)n - \gamma)p_S)\delta + G^2((3\gamma - 2)n - \gamma)(n-\gamma)\delta^2 > 0. \quad (31)$$

The inequalities (29) and (30) imply

$$n(1-\gamma)(\mu - d) - (1-G)((2-\gamma)n - \gamma))p_S > n(1-\gamma)G\delta > -(n-\gamma)(\mu - d) + ((2-\gamma)n - \gamma)Gp_S,$$

which implies (31) holds. In order to attract all sellers, M must therefore set $p_S < b - p_B$. This rules out the first possible deviation by M.

Now consider M's profit if it sets $p_B > \tilde{p}_B$ such that $0 < G < 1$ buyers join, and $p_S < b - p_B$ so sellers still join. Its profit is $G(p_S + p_B - k)q(p^* + p_B - b)$ where

$$q(p^* + p_B - b) = \frac{((n-\gamma)(\mu - d - Gp_S) + ((2-\gamma)n - \gamma - n(1-\gamma)G)(b - p_B))}{n((2-\gamma)n - \gamma)}.$$

We want to show that this profit is no larger than $(p_S + p_B - k)q(p^*(d + p_S + p_B - b))$. (Recall, we know from (21) that the profit $(p_S + p_B - k)q(p^*(d + p_S + p_B - b))$ is no more than M's profit in the proposed equilibrium in which all buyers join and it gets $(b - k)q(p^*(d))$ since $p_S < b - p_B$.) Note

$$(p_S + p_B - k)q(p^*(d + p_S + p_B - b)) = \frac{n-\gamma}{n((2-\gamma)n - \gamma)}(p_S + p_B - k)(\mu - d - p_S - p_B + b).$$

Thus, we need to show the following inequality holds

$$G((n-\gamma)(\mu - d - Gp_S) + ((2-\gamma)n - \gamma - n(1-\gamma)G)(b - p_B)) \leq (n-\gamma)(\mu - d - p_B - p_S + b).$$

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This inequality holds since

$$(n - \gamma)(\mu - d - p_S - p_B + b) - G((n - \gamma)(\mu - d - Gp_S) + ((2 - \gamma)n - \gamma - n(1 - \gamma)G)(b - p_B))$$

$$= (1 - G)((n - \gamma)(\mu - d) + (n - \gamma - n(1 - \gamma)G)(b - p_B) - (n - \gamma)(1 + G)p_S)$$

$$> (1 - G)((n - \gamma)(\mu - d) - ((2 - \gamma)n - \gamma)G(b - p_B))$$

$$> (1 - G)((n - \gamma)(\mu - d) - ((2 - \gamma)n - \gamma)(b - p_B))$$

$$> 0,$$

where the first inequality follows because $p_B + p_S < b$, the second inequality follows since $G < 1$, and the third inequality follows since $p_B > \tilde{p}_B > \bar{p}_B$, $\bar{p}_B = b - \frac{n - \gamma}{(2 - \gamma)n - \gamma}(\mu - d)$ and thus $b - p_B < b - \bar{p}_B = \frac{n - \gamma}{(2 - \gamma)n - \gamma}(\mu - d)$. This rules out the second possible deviation by M.

These findings imply the main results in Proposition 3 continue to hold in a setting with elastic aggregate demand even if we no longer assume that all buyers join in the equilibrium with price coherence, provided demand takes the linear form given in (28). With this demand specification, we have shown: M always imposes price coherence if permitted to do so; too few buyers join M without price coherence; too many buyers join M with price coherence; and consumer surplus increases when M operates without price coherence but decreases when M operates with price coherence, compared to the case in which M does not operate at all.

D   Imperfect competition between intermediaries

In this subsection, we formally prove the results given in Section 6.1 in the main text. We use the Perloff and Salop (1985) framework to model competition among N symmetric intermediaries. Each buyer incurs a cost c_i to join intermediary M^i, where c_i is i.i.d. and follows the same log-concave distribution $G(c)$ with $c \in [0, \bar{c}]$. Consistent with the cost faced by buyers, we consider a symmetric competitive bottleneck equilibrium in which each buyer joins one intermediary and all sellers join all intermediaries.

D.1   Without price coherence

Equilibrium: Consider the symmetric equilibrium. Buyers join M^i if $\beta^i - c_i \geq \max\{0, \max_{j \neq i}\{\beta^j - c_j\}\}$. Buyers join none of the intermediaries if $\max_j\{\beta^j - c_j\} < 0$. Consider the case where $\beta^i \geq 0$ for all i, otherwise there is no demand for M^i.

Online Appendix: Additional proofs and results

The demand for M^i is

$$D^i = \int_0^{\beta^i} \Pi_{j=1,j\neq i}^N (1 - G(\beta^j - \beta^i + c_i))dG(c_i).$$

M^i's profit is

$$(p_B^i + p_S^i - k^i)D^i.$$

Normalizing $p_B^i = 0$ and differentiating M^i's profit with respect to p_S^i and k^i gives

$$D^i + (p_S^i - k^i)\frac{dD^i}{dp_S^i} = 0,$$

$$-D^i - (p_S^i - k^i)\frac{dD^i}{dp_S^i}b'(k^i) = 0,$$

where

$$\frac{dD^i}{dp_S^i} = -\Pi_{j=1,j\neq i}^N (1 - G(\beta^j))g(\beta^i) - \int_0^{\beta^i} \sum_{j=1,j\neq i}^N g(\beta^j - \beta^i + c_i)\Pi_{l=1,l\neq i,j}^N (1 - G(\beta^l - \beta^i + c_i))dG(c_i).$$

In equilibrium, $b'(k^i) = 1$ so that $k^i = k^e$ for all i.

Imposing symmetry, so $\beta^i = \beta$ for all i, we have

$$D^i = D = \int_0^\beta (1 - G(c))^{N-1}dG(c) = \frac{1}{N} - \frac{1}{N}(1 - G(\beta))^N.$$

The first-order condition becomes

$$\frac{1}{N}(1 - (1 - G(\beta))^N) - (p_S - k^e)\left((1 - G(\beta))^{N-1}g(\beta) + (N-1)\int_0^\beta (1 - G(c))^{N-2}g(c)^2dc\right) = 0,$$

which implies

$$p_S^N = k^e + \frac{1 - (1 - G(\beta))^N}{N\left((1 - G(\beta))^{N-1}g(\beta) + (N-1)\int_0^\beta (1 - G(c))^{N-2}g(c)^2dc\right)}, \tag{32}$$

where p_S^N denotes the equilibrium price charged by the N intermediaries. Note $k^e < p_S^N < b(k^e)$, which establishes the first inequality in (4) in the main text.

Comparison with the monopoly case: From (2), the optimal p_S in the case of a monopoly intermediary (denoted p_S^m) solves

$$p_S^m = k^e + \frac{G(b(k^e) - p_S^m)}{g(b(k^e) - p_S^m)}.$$

To establish the second inequality in (4), we need to show $p_S^N < p_S^m$.

Suppose to the contrary that $p_S^N \geq p_S^m$. Then $\beta_N \equiv b(k^e) - p_S^N \leq b(k^e) - p_S^m \equiv \beta_m$. Then

$$
\begin{aligned}
p_S^m &= k^e + \frac{G(\beta_m)}{g(\beta_m)} \\
&\geq k^e + \frac{G(\beta_N)}{g(\beta_N)} \\
&> k^e + \frac{1 - (1 - G(\beta_N))^N}{N\left((1 - G(\beta_N))^{N-1}g(\beta_N) + (N-1)\int_0^{\beta_N}(1 - G(c))^{N-2}g(c)^2 dc\right)} \\
&= p_S^N.
\end{aligned}
$$

The first inequality comes from log-concavity of G, so that $\frac{G(c)}{g(c)}$ is increasing in c. The second inequality follows since

$$
\begin{aligned}
&G(\beta_N)\left(N(1 - G(\beta_N))^{N-1}g(\beta_N) + N(N-1)\int_0^{\beta_N}(1 - G(c))^{N-2}g(c)^2 dc\right) \\
&> N(1 - G(\beta_N))^{N-1}G(\beta_N)g(\beta_N) + N(N-1)g(\beta_N)\int_0^{\beta_N}(1 - G(c))^{N-2}G(c)g(c) dc \\
&= N(1 - G(\beta_N))^{N-1}G(\beta_N)g(\beta_N) - N(N-1)g(\beta_N)\int_0^{\beta_N}(1 - G(c))^{N-1}g(c) dc \\
&\quad + N(N-1)g(\beta_N)\int_0^{\beta_N}(1 - G(c))^{N-2}g(c) dc \\
&= N(1 - G(\beta_N))^{N-1}G(\beta_N)g(\beta_N) + (N-1)g(\beta_N)((1 - G(\beta_N))^N - 1) \\
&\quad - Ng(\beta_N)((1 - G(\beta_N))^{N-1} - 1) \\
&= g(\beta_N)(1 - (1 - G(\beta_N))^N),
\end{aligned}
$$

where the inequality comes again from log-concavity of G, so that $\frac{g(c)}{G(c)}$ is decreasing in c. This contradicts our assumption. As a result, $p_S^N < p_S^m$ and the second inequality in (4) holds.

The limit case ($N \to \infty$): Define $\underline{\beta} = \inf_N\{\beta_N\}$ and $\overline{\beta} = \sup_N\{\beta_N\}$. From above, we know $0 < \beta_m < \beta_N < b(k^e) - k^e$. Therefore, we have $0 < \underline{\beta} \leq \beta_N \leq \overline{\beta} \leq b(k^e) - k^e$ and $0 < G(\underline{\beta}) \leq G(\beta_N) \leq G(\overline{\beta}) < 1$. To establish (5) in the main text, we need to show $\lim_{N\to\infty} p_S^N = k^e$.

Since $G(c)$ is strictly increasing, twice-continuously differentiable and log-concave, we must have $g(c) > 0$ for $c \in (0, \bar{c})$. We consider two cases depending on whether $g(0) > 0$ or $g(0) = 0$. We add the following mild technical assumption: If $g(0) = 0$, then $\lim_{c\to 0} \frac{g'(c)}{g(c)}$ is unbounded. For example, the power function and Weibull distributions satisfy this assumption. We need no additional assumption if $g(0) > 0$, as is the case with the uniform distribution.

Online Appendix: Additional proofs and results

Case 1: $g(0) > 0$.

We have $\lim_{c \to 0} g(c) > 0$ because of the continuity of $g(c)$. Note

$$\lim_{N \to \infty} \left\{ \frac{1}{N} (1 - (1 - G(\beta_N))^N) \right\} = 0,$$

and

$$\lim_{N \to \infty} \left\{ (1 - G(\beta_N))^{N-1} g(\beta_N) \right\} = 0.$$

Moreover,

$$\lim_{N \to \infty} (N - 1) \int_0^{\beta_N} \left(1 - G(c) \right)^{N-2} g(c)^2 dc > 0,$$

since

$$\lim_{N \to \infty} (N - 1) \int_0^{\beta_N} \left(1 - G(c) \right)^{N-2} g(c)^2 dc$$

$$\geq \lim_{N \to \infty} (N - 1) \int_0^{\underline{\beta}} \left(1 - G(c) \right)^{N-2} g(c)^2 dc$$

$$= \lim_{N \to \infty} (N - 1) \int_0^{\delta} \left(1 - G(c) \right)^{N-2} g(c)^2 dc + \lim_{N \to \infty} \int_{\delta}^{\underline{\beta}} \left(1 - G(c) \right)^{N-2} g(c)^2 dc$$

$$\geq \xi \lim_{N \to \infty} (N - 1) \int_0^{\delta} \left(1 - G(c) \right)^{N-2} g(c) dc$$

$$= \xi \left(1 - \lim_{N \to \infty} (1 - G(\delta))^{N-1} \right) = \xi > 0.$$

The second inequality follows since, given the continuity of $g(c)$ and that $g(0) > 0$, there exists δ such that for $c \in [0, \delta)$, $g(c) \geq \xi > 0$. Applying these results to (32), we obtain $\lim_{N \to \infty} p_S^N = k^e$.

Case 2: $g(0) = 0$.

We have $\lim_{c \to 0} g(c) = 0$ because of the continuity of $g(c)$. Consider the denominator of (32). Using integration by parts and the fact that $g(0) = 0$, we obtain

$$N \left((1 - G(\beta_N))^{N-1} g(\beta_N) + (N - 1) \int_0^{\beta_N} (1 - G(c))^{N-2} g(c)^2 dc \right)$$

$$= \int_0^{\beta_N} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc.$$

Given that $\lim_{c \to 0} \frac{g'(c)}{g(c)}$ is positive and unbounded, for any $\xi > 0$, there exists $\delta \leq \beta_N$ such that $g'(c)/g(c) \geq \xi$ if $c \in (0, \delta)$. Thus,

$$\int_0^{\beta_N} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc$$

$$= \int_0^{\delta} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc + \int_{\delta}^{\beta_N} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc$$

$$= (1 - (1 - G(\delta))^N) \xi + \int_{\delta}^{\beta_N} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc. \tag{33}$$

Since

$$\lim_{N\to\infty} (1 - (1 - G(\delta))^N)\xi = \xi,$$

the limit of the first term in (33) is unbounded. Since $\left|\frac{g'(c)}{g(c)}\right|$ is bounded for $c > 0$, it can be shown that $\lim_{N\to\infty} \int_\delta^{\beta_N} N(1 - G(c))^{N-1} g(c) \frac{g'(c)}{g(c)} dc = 0$. Given the numerator of (32) satisfies

$$\lim_{N\to\infty} \left\{1 - (1 - G(\beta_N))^N\right\} = 1,$$

we have $\lim_{N\to\infty} p_S^N = k^e$, and (5) holds.

D.2 With price coherence

Equilibrium: Consider the symmetric equilibrium where all intermediaries impose price coherence and all sellers join all intermediaries and set a single price. Buyers join M^i if $b(k^i) - p_B^i - c_i \geq \max\left\{0, \max_{j\neq i}\{b(k^j) - p_B^j - c_j\}\right\}$. Buyers join none of the intermediaries if $\max_j\left\{b(k^j) - p_B^j - c_j\right\} < 0$.

The demand for M^i is

$$D^i = \int_0^{b(k^i) - p_B^i} \Pi_{j=1, j\neq i}^N (1 - G(b(k^j) - p_B^j - b(k^i) + p_B^i + c_i)) dG(c_i)$$

and M^i's profit is

$$(p_B^i + p_S^i - k^i) D^i.$$

In the symmetric equilibrium where all sellers join all intermediaries, they set the equilibrium price $\hat{p} = d + \sum_{j=1}^N D^j p_S^j + t$, where $t = \frac{1}{\sigma n}$. They obtain the standard seller profit $\frac{t}{n}$ (i.e. $\frac{1}{\sigma n^2}$).

Suppose seller i deviates by joining none of the intermediaries. Its deviation profit is

$$\max_{p_i'} \left\{ \begin{array}{c} \sum_{j=1}^N D^j (p_i' - d)\left(\frac{1}{n} + \sigma\left(\hat{p} - p_i' - b(k^j) + p_B^j\right)\right) \\ + \left(1 - \sum_{j=1}^N D^j\right)(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i')\right) \end{array} \right\},$$

which can be written as

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma\left(d + t - p_i' - \sum_{j=1}^N D^j \beta^j\right)\right) \right\}.$$

The deviation profit is no larger than the standard seller profit $\frac{t}{n}$ if and only if $\sum_{j=1}^N D^j \beta^j \geq 0$. Thus, no seller can profitably deviate by not joining any intermediary if $\sum_{j=1}^N D^j \beta^j \geq 0$.

Suppose seller i deviates by not joining $L < N$ intermediaries (say from M^1 to M^L). Seller i's

deviation profit is then

$$\max_{p_i'} \left\{ \begin{array}{l} \sum_{j=1}^{L} D^j (p_i' - d) \left(\frac{1}{n} + \sigma(\hat{p} - p_i' - b(k^j) + p_B^j) \right) \\ + \sum_{j=L+1}^{N} D^j (p_i' - d - p_S^j) \left(\frac{1}{n} + \sigma(\hat{p} - p_i') \right) \\ + \left(1 - \sum_{j=1}^{N} D^j \right) (p_i' - d) \left(\frac{1}{n} + \sigma(\hat{p} - p_i') \right) \end{array} \right\}.$$

The optimal deviation price is

$$p_i' = d + t + \sum_{j=L+1}^{N} D^j p_S^j - \frac{1}{2} \sum_{j=1}^{L} D^j \beta^j,$$

and

$$\hat{p} - p_i' = \frac{1}{2} \sum_{j=1}^{L} D^j (b(k^j) - p_B^j + p_S^j).$$

Note seller i's market share from buyers who join M^l, where $l = 1, \dots, L$, is

$$\frac{1}{n} + \sigma \left(\frac{1}{2} \sum_{j=1}^{L} D^j (b(k^j) - p_B^j + p_S^j) - b(k^l) + p_B^l \right)$$

$$> \frac{1}{n} + \sigma \left(\frac{1}{2} D^l (b(k^l) - p_B^l + p_S^l) - b(k^l) + p_B^l \right)$$

$$> \frac{1}{n} - \sigma b(\bar{k}) \left(1 - \frac{D^l}{2} \right),$$

which is positive given $t = \frac{1}{\sigma n} > b(\bar{k})$.

Therefore, the deviation profit is

$$\frac{1}{\sigma n^2} \left(1 - \frac{\sigma n \sum_{j=1}^{L} D^j \beta^j}{2} \right)^2 - \sigma \sum_{j=1}^{L} D^j (b(k^j) - p_B^j) \left(\sum_{j=L+1}^{N} D^j p_S^j \right).$$

This is strictly less than the standard seller profit $\frac{1}{\sigma n^2}$, given $\beta^j \geq 0$, where $j = 1, 2, \dots, L$.

Now consider the intermediaries' choices. To focus on the symmetric equilibrium, suppose all M^j where $j \neq i$ set $\beta^j = 0$. Then to keep sellers joining, M^i needs to set $\beta^i \geq 0$. Subject to this constraint, its profit is

$$(p_B^i + p_S^i - k^i) D^i.$$

Since D^i is decreasing in p_B^i but not p_S^i, M^i does best setting $p_B^i = 0$ and $p_S^i = b(k^i)$. Thus M^{i}'s profit is

$$(b(k^i) - k^i) \int_0^{b(k^i)} \Pi_{j=1, j \neq i}^{N} (1 - G(b(k^j) - b(k^i) + c_i)) dG(c_i).$$

Differentiating with respect to k^i gives

$$(b'(k^i) - 1) D^i + (b(k^i) - k^i) \frac{dD^i}{dk^i} = 0,$$

where

$$\frac{dD^i}{dk^i} = b'(k^i)\Big(\Pi^N_{j=1,j\neq i}(1 - G(b(k^j)))g(b(k^i))$$

$$+ \int_0^{b(k^i)} \sum_{j=1,j\neq i}^N g(b(k^j) - b(k^i) + c_i)\Pi^N_{l=1,l\neq i,j}(1 - G(b(k^l) - b(k^i) + c_i))dG(c_i)\Big).$$

Imposing the symmetry condition that $k^i = k$ for all i,

$$D^i = D = \int_0^{b(k)}(1 - G(c))^{N-1}dG(c) = \frac{1}{N} - \frac{1}{N}(1 - G(b(k)))^N$$

and

$$\frac{dD}{dk} = b'(k)(1 - G(b(k)))^{N-1}g(b(k)) + (N-1)\int_0^{b(k)}(1 - G(c))^{N-2}g(c)^2dc.$$

The first-order condition becomes

$$(b'(k_N) - 1)\frac{1}{N}(1 - (1 - G(b(k_N)))^N)$$

$$+(b(k_N) - k_N)b'(k_N)\left((1 - G(b(k_N)))^{N-1}g(b(k_N)) + (N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right) = 0,$$

where k_N denotes the equilibrium with N intermediaries. Note that $k^e < k_N < \bar{k}$, which establishes the second inequality in (7) in the main text.

Comparison with the monopoly case: Recall that the optimal k^* in the case of a monopoly intermediary maximizes

$$(b(k) - k)G(b(k)),$$

which implies the following first consider condition

$$(b'(k^*) - 1)G(b(k^*)) + (b(k^*) - k^*)b'(k^*)g(b(k^*)) = 0.$$

It implies that

$$\frac{(b(k^*) - k^*)b'(k^*)}{1 - b'(k^*)} = \frac{G(b(k^*))}{g(b(k^*))},$$

where $k^e < k^* < \bar{k}$. To establish the first inequality in (7), we need to show $k_N > k^*$.

We prove this by contradiction. Define $f(k) = \frac{(b(k)-k)b'(k)}{1-b'(k)}$, which satisfies $f'(k) < 0$ for $k^e < k < \bar{k}$. We have

$$f(k_N) = \frac{1 - (1 - G(b(k_N)))^N}{N\left((1 - G(b(k_N)))^{N-1}g(b(k_N)) + (N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right)},$$

$$f(k^*) = \frac{G(b(k^*))}{g(b(k^*))}.$$

Suppose $k_N \leq k^*$. Since $b(k)$ is increasing in k, $b(k_N) \leq b(k^*)$. Following the same argument as the

Online Appendix: Additional proofs and results

case without price coherence,

$$f(k_N) = \frac{1 - (1 - G(b(k_N)))^N}{N\left((1 - G(b(k_N)))^{N-1}g(b(k_N)) + (N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right)}$$
$$< \frac{G(b(k_N))}{g(b(k_N))}$$
$$\leq \frac{G(b(k^*))}{g(b(k^*))} = f(k^*),$$

or $k_N > k^*$. This contradicts $k_N \leq k^*$. As a result $k_N > k^*$ and the first inequality in (7) holds.

The limit case $(N \to \infty)$:

To establish (8) in the main text, we need to show that

$$\lim_{N\to\infty}\{b(k_N) - k_N\} = 0$$

or $\lim_{N\to\infty} k_N = \overline{k}$.

We have

$$\lim_{N\to\infty}\left\{(b'(k) - 1)\frac{1}{N}(1 - (1 - G(b(k_N)))^N)\right\} = 0,$$

$$\lim_{N\to\infty}\left\{(b(k_N) - k_N)b'(k_N)(1 - G(b(k_N)))^{N-1}g(b(k_N))\right\} = 0,$$

given $0 < G(b(k_N)) < 1$ for $b(k^e) < b(k_N) < b(\overline{k})$. We also have

$$\lim_{N\to\infty}\left\{(b(k_N) - k_N)b'(k_N)(N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right\} = 0$$

so that the first-order condition holds.

Since

$$\lim_{N\to\infty}\left\{(b(k_N) - k_N)b'(k_N)(N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right\}$$

$$= \lim_{N\to\infty}\left\{(b(k_N) - k_N)b'(k_N)\right\}\lim_{N\to\infty}\left\{(N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right\}$$

and

$$\lim_{N\to\infty}\left\{(N-1)\int_0^{b(k_N)}(1 - G(c))^{N-2}g(c)^2dc\right\} > 0$$

following the same logic as before, we must have

$$\lim_{N\to\infty}\left\{(b(k_N) - k_N)b'(k_N)\right\} = 0,$$

which implies

$$\lim_{N\to\infty}\{b(k_N) - k_N\} = 0$$

given $b'(k_N) > 0$ for $k^e < k_N < \overline{k}$.

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Therefore, $\lim_{N \to \infty} k_N = \bar{k}$, and (8) holds.

E   Usage heterogeneity

In this subsection, we formally prove the results given in Section 6.2 in the main text. We modify the model of Section 5 in the main text by assuming the benefit of *using M* is $b(k) - c$, where buyers draw c from the distribution G as before. Since a buyer can only use one the two identical competing intermediaries for a given purchase, assume that each buyer continues to join at most one intermediary. We also adopt the tie-breaking rule, that a buyer only joins an intermediary if the buyer plans to use it. We additionally assume $b(k^e) - k^e > E[c|c < b(k^e)]$, so that the net welfare created by intermediaries at the efficient investment level is positive even when buyers do not have to pay to use their services. All other assumptions are as in Section 5.

E.1   Without price coherence

The proof is identical to the proof of Proposition 4 in the main text.

E.2   With price coherence

Each M^i $(i = 1, 2)$ sets $0 \leq k^i \leq \bar{k}$, $p_B^i \geq 0$ and $p_S^i \geq 0$ in stage 1. Define $\hat{c}^i = E[c|c < b(k^i) - p_B^i]$. Then define $\hat{\beta}^i = b(k^i) - p_B^i - p_S^i - \hat{c}^i$. This is the expected net surplus from usage of M^i given buyers want to use M^i with price coherence.

Consider buyers' and sellers' equilibrium strategies in stage 2. These are: (i) if $\hat{\beta}^i \geq 0$ and $b(k^i) - p_B^i > b(k^j) - p_B^j$, then buyers only join M^i (doing so if they draw $c \leq b(k^i) - p_B^i$) and sellers join both intermediaries and set the price $\hat{p}^i = d + G(b(k^i) - p_B^i)p_S^i + t$; (ii) if $\hat{\beta}^i \geq 0$, $\hat{\beta}^j < 0$ and $b(k^i) - p_B^i \leq b(k^j) - p_B^j$, then the same strategies hold as in (i) except sellers only join M^i; (iii) if $\hat{\beta}^1 \geq 0$, $\hat{\beta}^2 \geq 0$ and $b(k^1) - p_B^1 = b(k^2) - p_B^2$, which can be denoted $b(k) - p_B$, then buyers join an intermediary if they draw $c \leq b(k) - p_B$, randomizing over which one, while all sellers join both intermediaries and set the price $\hat{p} = d + G(b(k) - p_B)\left(\frac{p_S^1}{2} + \frac{p_S^2}{2}\right) + t$; (iv) if $\hat{\beta}^1 < 0$ and $\hat{\beta}^2 < 0$, then no buyers or sellers join either intermediary, and each seller prices at $d + t$.

We show that (i)-(iv) characterize an equilibrium in the stage 2 subgame. Consider case (i). Buyers strictly prefer to join M^i over M^j, and do so whenever they draw $c \leq b(k^i) - p_B^i$. Consider sellers' optimal strategies. If they all join M^i, then buyers that join M^i always want to make use of it, and sellers set the price $\hat{p}^i = d + G(b(k^i) - p_B^i)p_S^i + t$ in equilibrium, obtaining the standard seller profit.

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Now consider a seller i that deviates and does not join M^i and sets the price p_i'. Its deviation profit is

$$\max_{p_i'} \left\{ \begin{array}{c} G(b(k^i) - p_B^i)(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p}^i - p_i' - (b(k^i) - p_B^i - \hat{c}^i)))\right) \\ +(1 - G(b(k^i) - p_B^i))(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p}^i - p_i')\right) \end{array} \right\}.$$

Note that the deviating seller attracts some buyers that join M^i and not all buyers that do not join M^i given our assumption that $t = \frac{1}{\sigma n} > b(\overline{k})$. Then the deviation profit can be rewritten as

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p}^i - p_i' - (b(k^i) - p_B^i - \hat{c}^i)G(b(k^i) - p_B^i))\right) \right\},$$

which after substituting in $\hat{p}^i$ can be written as

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma(d + t - p_i' - \hat{\beta}^i)\right) \right\}.$$

Thus, sellers are willing to join M^i provided $\hat{\beta}^i \geq 0$. Sellers also join M^j, given that they cannot do better by not joining M^j. This establishes (i).

Consider case (ii). Suppose there is an equilibrium where sellers join M^j. Then buyers prefer to join and use M^j. But then since $\hat{\beta}^j < 0$, sellers prefer not to join M^j, contradicting that this is an equilibrium. Therefore, the only equilibrium is where sellers join M^i. With sellers only joining M^i, the same type of analysis as case (i) applies. Sellers join provided that their buyers' expected net benefit per transaction of being able to use M exceeds the fee the seller pays per transaction. Since buyers use M whenever they draw $c < b(k^i) - p_B^i$, this is true provided $\hat{\beta}^i \geq 0$. This establishes (ii).

Consider case (iii). Given that sellers join both intermediaries, buyers that join an intermediary are indifferent between joining M^1 and M^2 because they face the same benefits from going through each intermediary and the same seller price due to price coherence. They therefore cannot do better than to randomize between the two intermediaries, implying each intermediary attracts half the buyers that join an intermediary. Note that we can define $\hat{c} = \hat{c}^1 = \hat{c}^2$ for this case. Given that all other sellers have joined both intermediaries and price at $\hat{p}$, if seller i does likewise, it obtains the standard seller profit. If it instead deviates and does not join either intermediary and sets its price to be p_i', its deviation profit is

$$\max_{p_i'} \left\{ \begin{array}{c} G(b(k) - p_B)(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i' - (b(k) - p_B - \hat{c})))\right) \\ +(1 - G(b(k) - p_B))(p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i')\right) \end{array} \right\}$$

or

$$\max_{p_i'} \left\{ (p_i' - d)\left(\frac{1}{n} + \sigma(\hat{p} - p_i' - G(b(k) - p_B)(b(k) - p_B - \hat{c}))\right) \right\}$$

Substituting in $\hat{p}$ from (iii), its deviation profit can be written as

$$\max_{p'_i} \left\{ (p'_i - d) \left(\frac{1}{n} + \sigma \left(d + t - p'_i - G(b(k) - p_B) \left(\frac{\hat{\beta}^1 + \hat{\beta}^2}{2} \right) \right) \right) \right\}. \tag{34}$$

Compare the profit in (34) with the standard seller problem. The profit expression is the same, with the same margin and demand at every price if $\hat{\beta}^1 + \hat{\beta}^2 = 0$, but lower demand if $\hat{\beta}^1 + \hat{\beta}^2 > 0$. Thus, since the profit in the standard seller problem is the same as the profit in the proposed equilibrium (i.e. standard seller profit), each seller i does not want to deviate in this way given $\hat{\beta}^1 \geq 0$ and $\hat{\beta}^2 \geq 0$ and that buyers and the other sellers follow their equilibrium strategies.

The alternative deviation is that seller i only joins one intermediary, say M^1, and sets its price to be p'_i for buyers purchasing through M^1 or directly. Then seller i's deviation profit is

$$\max_{p'_i} \left\{ \begin{array}{l} \frac{1}{2}G(b(k) - p_B)(p'_i - d) \left(\frac{1}{n} + \sigma(\hat{p} - p'_i - (b(k) - p_B - \hat{c})) \right) \\ + \frac{1}{2}G(b(k) - p_B)(p'_i - d - p_S^1) \left(\frac{1}{n} + \sigma(\hat{p} - p'_i) \right) \\ + (1 - G(b(k) - p_B))(p'_i - d) \left(\frac{1}{n} + \sigma(\hat{p} - p'_i) \right) \end{array} \right\}. \tag{35}$$

Substituting the expression for $\hat{p}$ from (iii) above into (35) and solving for the optimal p'_i implies

$$p'_i = d + t + \frac{1}{2}G(b(k) - p_B)p_S^1 - \frac{1}{4}G(b(k) - p_B)\hat{\beta}^2.$$

Note

$$\hat{p} - p'_i = \frac{1}{4}(b(k) - p_B - \hat{c} + p_S^2)G(b(k) - p_B)$$

and the deviation profit equals

$$\frac{1}{32t} \left(16t^2 + G(b(k) - p_B) \left(G(b(k) - p_B)(\hat{\beta}^2)^2 - 8t\hat{\beta}^2 - 4G(b(k) - p_B)(b(k) - p_B - \hat{c})p_S^1 \right) \right).$$

The deviating seller still attracts some buyers that join M^i given our assumption that $t = \frac{1}{\sigma n} > b(\bar{k})$. The difference between the resulting deviation profit and the standard seller profit is

$$\frac{G(b(k) - p_B)}{32t} \left((\hat{\beta}^2)^2 G(b(k) - p_B) - 8t\hat{\beta}^2 - 4(b(k) - p_B - \hat{c})p_S^1 G(b(k) - p_B) \right). \tag{36}$$

Given $t > b(\bar{k}) > \hat{\beta}^2 \geq 0$ and given $b(k) - p_B \geq \hat{c}$, the expression in (36) is strictly negative. Thus, seller i is strictly worse off deviating in this way, which establishes (iii).

Consider case (iv). The same analysis as above implies that there is no equilibrium in which all sellers join both intermediaries. The trivial equilibrium is selected instead in which buyers and sellers do not join either intermediary. The standard seller problem applies, and sellers price at $d + t$ in equilibrium. This establishes (iv).

In stage one, each M^i only expects to receive positive demand if $b(k^i) - p_B^i \geq b(k^j) - p_B^j$ and

$\hat{\beta}^i \geq 0$. Since starting from $b(k^1) - p_B^1 = b(k^2) - p_B^2$, M^i can attract all buyers that join rather than half of them if it increases $b(k^i) - p_B^i$ by an infinitesimal amount, it always wants to do so provided its margin remains positive. As a result, in equilibrium, M^i offers the maximum possible $b(k^i) - p_B^i$ consistent with cost recovery (i.e. $p_B^i + p_S^i \geq k^i$) and $\hat{\beta}^i = b(k^i) - p_B^i - p_S^i - E[c|c < b(k^i) - p_B^i] \geq 0$.

To characterize the solution, we first show that $p_B^i + p_S^i = k^i$. If $p_B^i + p_S^i > k^i$, then it would be possible to lower p_B^i or raise k^i and therefore increase $b(k^i) - p_B^i$ without violating the two constraints. This requires that $\hat{\beta}^i$ is increasing in $b(k^i) - p_B^i$, or equivalently that

$$v(b) = \frac{\int_0^b (b - c) dG(c)}{G(b)}$$

is increasing in b. Using integration by parts, this can be rewritten as

$$v(b) = \frac{y(b)}{y'(b)}$$

where $y(b) = \int_0^b G(c) dc$. Given G is log-concave, $y(b)$ is log-concave from Theorem 1 in Bagnoli and Bergstrom (2005). Therefore $v(b)$ is increasing in b, as required.

Substituting $p_B^i + p_S^i = k^i$ into the constrained maximization problem above, it becomes

$$\max_{k^i, p_B^i} \{ b(k^i) - p_B^i \}$$

s.t.

$$b(k^i) - k^i - E[c|c < b(k^i) - p_B^i] \geq 0.$$

Note that the solution cannot involve $p_B^i > 0$. If it did, then M^i can lower p_B^i to zero and lower k^i so that $b(k^i)$ is also lowered by the same amount. This would not change $b(k^i) - p_B^i$, but would raise $b(k^i) - k^i$ and relax the constraint provided k^i exceeds k^e which is true given our assumption $b(k^e) - E[c|c < b(k^e)] > k^e$. Thus, the problem simplifies to

$$\max_{k^i} \{ b(k^i) \}$$

s.t.

$$b(k^i) - k^i - E[c|c < b(k^i)] \geq 0.$$

The constraint is decreasing in k^i for $k^i \geq k^e$. Since the objective function is increasing in k^i, the solution is where the constraint is just binding, which establishes (9) in the main text. This also implies $p_S^i = b(k^i) - E[c|c < b(k^i)]$.

Both intermediaries choose the same fees and investment levels in equilibrium, and intermediaries make zero profit. Define $\hat{c}(k) = E[c|c < b(k)]$. Then $b(\bar{k}) - \hat{c}(\bar{k}) < \bar{k}$ by definition. From our earlier

assumption $b(k^e) - \hat{c}(k^e) > k^e$. Then since $b(k) - k - \hat{c}(k)$ is decreasing in k for $k > k^e$ (which follows since $b'(k) - 1 - d\hat{c}(k)/dk < 0$ for $k > k^e$), there exists a unique level of k_U which solves (9) in the main text, and it is such that $k^e < k_U < \overline{k}$. This establishes (10) in the main text.

F   Membership fees

In this subsection, we formally prove the results given in Section 6.3 in the main text. Consider the case of a single intermediary M that can observe buyers joining costs c and charge membership fees.

Without price coherence, suppose that M extracts the maximum possible amount from each buyer $(b(k) - c)$. M serves buyers with c from 0 to y, and chooses k and y. Then M's profit is

$$\int_0^y (b(k) - k - c)dG(c).$$

First order conditions imply the optimal outcome $y = b(k) - k$ and $b'(k) - 1 = 0$. This establishes the result in (11) in the main text.

With price coherence, M's profit is

$$\int_0^y (2b(k) - k - c)dG(c)$$

reflecting the additional fee of $b(k)$ that M collects from sellers. First order conditions imply the optimal outcome $y = 2b(k) - k$ and $2b'(k) - 1 = 0$. M sets $k = k_M$. Since $k_M > k^e$, this establishes the result in (12) in the main text.

Consider the case with two identical competing intermediaries M^1 and M^2. The analysis of the resulting competitive bottleneck equilibrium is similar to Section 5 in the main text. Competition is Bertrand-like: The intermediary that offers buyers the highest surplus takes the whole market.

Without price coherence, intermediaries do not charge membership fees in equilibrium, since any intermediary that did could be undercut by another that charged a lower membership fee, thereby attracting all buyers and sellers.

With price coherence, the same analysis of the competitive bottleneck equilibrium in Section 5.2 applies, and the intermediary that can offer buyers the highest surplus takes the whole market. Given this, an intermediary would only use positive membership fees to buyers if doing so allows it to set a higher level of k such that buyers are better off. Consider the equilibrium level of k in the absence of membership fees from Section 5.2, which is $\overline{k}$. If M^i offers $k^i = \overline{k} + \delta$ for some $\delta > 0$, then it can charge a membership fee $F^i = \overline{k} + \delta - b(\overline{k} + \delta)$ to each buyer it attracts while still breaking even. This creates a profitable deviation if this also increases buyer surplus, since it can then charge

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a slightly higher membership fee and still attract all buyers. Buyer surplus with this deviation would be $b(\overline{k} + \delta) - F^i = 2b(\overline{k} + \delta) - (\overline{k} + \delta)$.

There are two cases to consider.

(i) Suppose $k_M > \overline{k}$. This arises when the buyer-side benefit function $b(k)$ is sufficiently steep at the point where the overall net benefits of investment become negative. In particular, it requires that at the point where the overall net benefit of investment becomes negative, an extra \$1 of investment generates more than \$0.50 in buyer-side benefits. Then $2b(k_M) - k_M > 2b(\overline{k}) - \overline{k} = b(\overline{k})$ and buyer surplus can be increased through the use of a membership fee and an investment level of $k > \overline{k}$. The equilibrium occurs when both intermediaries set the investment level k_M, since this maximizes $2b(\overline{k} + \delta) - (\overline{k} + \delta)$. The equilibrium investment level k_M exceeds that without membership fees $(\overline{k})$, so there is an even greater level of over-investment in buyer-side benefits; the net benefit generated by intermediaries is negative. Buyers join provided $c < b(k_M) - F^i$, so even more buyers join compared to the case without membership fees given $b(k_M) - F^i = 2b(k_M) - k_M > b(\overline{k})$. The equilibrium membership fee in this case equals $k_M - b(k_M)$, which is the same for all buyers despite the ability of intermediaries to set different fees for different buyers. Given that intermediaries make zero profit in equilibrium, eliminating price coherence increases consumer surplus by $\pi^e G(\pi^e) + (k_M - b(k_M))G(2b(k_M) - k_M) + \int_{\pi^e}^{2b(k_M) - k_M} c\, dG(c)$. Eliminating intermediation increases consumer surplus and welfare by $(k_M - b(k_M))G(2b(k_M) - k_M) + \int_0^{2b(k_M) - k_M} c\, dG(c)$. The extra term in consumer surplus and welfare compared to that given in Proposition 6 is the loss $(k_M - b(k_M))G(2b(k_M) - k_M)$. This is the surplus destroyed because the investment level is set so high that net benefits created by intermediaries are negative. In addition, there are joining costs incurred by buyers that should not join an intermediary in the efficient outcome. These costs increase with membership fees because even more buyers join. Thus, in case $k_M > \overline{k}$, the results of Proposition 6 are even more pronounced when M can set membership fees to extract buyer surplus.

(ii) Suppose $k_M \le \overline{k}$. This arises when the buyer-side benefit function $b(k)$ is sufficiently flat at the point where the overall net benefits of investment become negative. Specifically, it arises when at the point where the overall net benefit of investment becomes negative, an extra \$1 of investment generates less than \$0.50 in buyer-side benefits. Then $2b(\overline{k} + \delta) - (\overline{k} + \delta) \le 2b(\overline{k}) - \overline{k} = b(\overline{k})$ and buyer surplus cannot be increased through the use of a membership fee and a higher level of investment k. The equilibrium characterized in Proposition 5 therefore remains an equilibrium and membership fees would be set to zero. In this case, the findings of Section 5.2 in which we assumed away membership fees continue to hold even if intermediaries can use membership fees.

G   Asymmetric sellers

To allow for asymmetric outcomes we adopt the spokes model of seller competition (Chen and Riordan, 2007) since it generalizes the "Hotelling" model in a way that can capture non-localized seller competition. It is a special case of our more general formulation in (1) in the main text. Specifically, suppose there are n sellers, and the total market share for seller i is

$$s_i = \frac{2}{n(n-1)} \sum_{j=1, j \neq i}^{n} \left(\frac{1}{2} + \frac{w_j - w_i}{2t} \right) = \frac{2}{n(n-1)} \sum_{j=1, j \neq i}^{n} s_{ij},$$

where w_i is the surplus offered by seller i to a buyer and s_{ij} is the market share of seller i of the buyers located between seller i and seller j.

Suppose there exists an alternative seller (denoted a) for which buyers never obtain any benefit $b(k)$ when they purchase through M. This could be because it is a maverick firm that never joins M, or it is a no-frills seller that has no incentive to join M since buyers are assumed to obtain no benefit when they buy from it through M. All other sellers are modeled as before. We look for an asymmetric equilibrium in which all sellers except for the alternative seller join M. Note that unlike the symmetric case, the measure of buyers joining M not only depends on $b - p_B$ but also on p_S. For brevity, we suppress the arguments for $b(k)$ and $G(c)$ in the analysis that follows, denoting $b \equiv b(k)$ and $G \equiv G(c)$. Other notation remains unchanged. To focus on the interior solution so that all sellers capture positive markets on both types of buyers, assume that t is sufficiently large.

G.1   Without price coherence

The profit of the alternative seller a is

$$\pi_a = \left(G \frac{2}{n(n-1)} \sum_{j=1, j \neq a}^{n} \left(\frac{1}{2} + \frac{p_j^m - (b - p_B) - p_a}{2t} \right) \right.$$
$$\left. + (1 - G) \frac{2}{n(n-1)} \sum_{j=1, j \neq a}^{n} \left(\frac{1}{2} + \frac{p_j^d - p_a}{2t} \right) \right) (p_a - d),$$

where p_j^d is the price set by seller j for a direct purchase, p_j^m is the price set by seller j for a purchase through M, and p_a is the price set by seller a (for a direct purchase). In comparison, the profit of

seller $i \neq a$ is

$$
\begin{aligned}
\pi_i = & G \frac{2}{n(n-1)} \left(\left(\frac{1}{2} + \frac{p_a + b - p_B - p_i^m}{2t} \right) + \sum_{j=1, j \neq a,i}^{n} \left(\frac{1}{2} + \frac{p_j^m - p_i^m}{2t} \right) \right) (p_i^m - p_S - d) \\
& + (1 - G) \frac{2}{n(n-1)} \left(\left(\frac{1}{2} + \frac{p_a - p_i^d}{2t} \right) + \sum_{j=1, j \neq a,i}^{n} \left(\frac{1}{2} + \frac{p_j^d - p_i^d}{2t} \right) \right) (p_i^d - d).
\end{aligned}
$$

The corresponding first order conditions of the sellers are

$$
(n-1)(t+d) - (n-1)G(b - p_B) + G \sum_{j \neq a}^{n} p_j^m + (1 - G) \sum_{j \neq a}^{n} p_j^d - 2(n-1)p_a = 0,
$$

$$
(n-1)(t + d + p_S) + p_a + b - p_B + \sum_{j \neq a,i}^{n} p_j^m - 2(n-1)p_i^m = 0,
$$

$$
(n-1)(t+d) + p_a + \sum_{j \neq a,i}^{n} p_j^d - 2(n-1)p_i^d = 0.
$$

Imposing the symmetry condition, $p_j^d = p^d$ and $p_j^m = p^m$ for $j \neq a$, we have the equilibrium prices

$$
p^d = t + d - \frac{G(n-1)\beta}{n(2n-1)},
$$

$$
p^m = t + d + p_S + \frac{1}{n} \left(1 - \frac{G(n-1)}{2n-1} \right) \beta,
$$

$$
p_a = t + d - \frac{G(n-1)\beta}{2n-1},
$$

where $\beta = b - p_B - p_S$. The market shares of seller a with respect to each other firm for buyers that join M and buyers that do not, along with market shares between any two other firms of each type of buyer, are as follows:

$$
s_{ai}^m = \frac{1}{2} - \frac{(n-1)\left(1 - \frac{G(n-1)}{2n-1} \right) \beta}{2nt},
$$

$$
s_{ai}^d = \frac{1}{2} + \frac{G(n-1)^2 \beta}{2n(2n-1)t},
$$

$$
s_{ij}^m = s_{ij}^d = \frac{1}{2},
$$

where the superscript m denotes the buyers who have joined M and d denotes those who have not. Denoting the equilibrium profit for seller a as π_a^* and for all other sellers as π_i^*, we have

$$
\pi_a^* = \frac{t}{n} \left(1 - \frac{G(n-1)\beta}{(2n-1)t} \right)^2,
$$

$$
\pi_i^* = \frac{Gt}{n} \left(1 + \frac{1}{n} \left(1 - \frac{G(n-1)}{(2n-1)} \right) \frac{\beta}{t} \right)^2 + \frac{(1-G)t}{n} \left(1 - \frac{G(n-1)\beta}{n(2n-1)t} \right)^2.
$$

Provided $\beta \geq 0$, one can check that sellers do not want to deviate, and in particular sellers do not want to abandon M.

We now turn to the decision of buyers on whether to join M. The expected surplus a buyer gets from joining M is

$$CS^m = v - \frac{2}{n}\left(s_{ai}^m\left(\frac{t}{2}s_{ai}^m + p_a\right) + (1 - s_{ai}^m)\left(\frac{t}{2}(1 - s_{ai}^m) + p^m - b + p_B\right)\right)$$
$$- \frac{n-2}{n}\left(\frac{t}{2}s_{ij}^m + p^m - b + p_B\right)$$

and the expected surplus of not joining M is

$$CS^d = v - \frac{2}{n}\left(s_{ai}^d\left(\frac{t}{2}s_{ai}^d + p_a\right) + (1 - s_{ai}^d)\left(\frac{t}{2}(1 - s_{ai}^d) + p^d\right)\right) - \frac{n-2}{n}\left(\frac{t}{2}s_{ij}^d + p^d\right).$$

The expected gain from joining M is

$$CS^m - CS^d = -\frac{2}{n}\left((s_{ai}^m - s_{ai}^d)\left(t(s_{ai}^m + s_{ai}^d - 1) + p_a\right) + (1 - s_{ai}^m)(p^m - b + p_B) - (1 - s_{ai}^d)p^d\right)$$
$$+ \frac{n-2}{n}(p^d - p^m + b - p_B)$$
$$= -\frac{(n-1)^2\beta^2}{2n^3t}\left(-1 + \frac{2(n-1)G}{2n-1}\right) + \frac{(n-1)^2\beta}{n^2}.$$

Buyers join if and only if $c \leq \tilde{c}$, where $\tilde{c} = CS^m - CS^d$.

Given $G(\tilde{c})$ of buyers join M, M's demand is

$$D^m = G(\tilde{c})\left(\frac{2}{n}(1 - s_{ai}^m) + \frac{n-2}{n}\right),$$

and it obtains the profit

$$(p_B + p_S - k)D^m. \tag{37}$$

Note it is straightforward to confirm these profits only depend on $p_B + p_S$. We therefore set $p_B = 0$.

In general, consumer surplus with intermediation is

$$CS^M = G(\tilde{c})CS^m + (1 - G(\tilde{c}))CS^d - \int_0^{\tilde{c}} c\, dG(c)$$

and consumer surplus without intermediation is

$$CS^N = v - \frac{5t}{4} - d,$$

so that the change of consumer surplus due to M is

$$\Delta CS = CS^M - CS^N.$$

The welfare contribution of M can be obtained by summing ΔCS, M's profit in (37) and the change in sellers' profit $\pi_a^* + (n-1)\pi_i^* - t$.

Large number of sellers

Consider the limit as $n \to \infty$. Then $\tilde{c}$ converges to β. The demand for M converges to $G(\beta)$.

Online Appendix: Additional proofs and results

Thus, M chooses p_S and k to maximize

$$(p_S - k)G(b(k) - p_S).$$

The result converges to the case in which all sellers join in Section 4.1.

G.2 With price coherence

To characterize the asymmetric equilibrium, we focus on the case where $b, p_B, p_S \geq 0$ and $b - p_B > 0$. (Otherwise no buyers join M.) We focus on the case with an interior solution so that all sellers attract positive shares of both types of buyers given that t is sufficiently large. Consider the equilibrium in which all sellers except for the alternative seller a join M. Then the profit of seller a is

$$\pi_a = \left(G \frac{2}{n(n-1)} \sum_{j=1, j \neq a}^{n} \left(\frac{1}{2} + \frac{p_j - (b - p_B) - p_a}{2t} \right) \right.$$

$$\left. + (1-G) \frac{2}{n(n-1)} \sum_{j=1, j \neq a}^{n} \left(\frac{1}{2} + \frac{p_j - p_a}{2t} \right) \right) (p_a - d),$$

while the profit of seller $i \neq a$ is

$$\pi_i = G \frac{2}{n(n-1)} \left(\left(\frac{1}{2} + \frac{p_a + b - p_B - p_i}{2t} \right) + \sum_{j=1, j \neq a,i}^{n} \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right) \right) (p_i - p_S - d)$$

$$+ (1-G) \frac{2}{n(n-1)} \sum_{j=1, j \neq i}^{n} \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right) (p_i - d).$$

The first-order condition from seller a's maximizing choice of p_a is

$$(n-1)t + \sum_{j=1, j \neq a}^{n} (p_j - p_a) - (n-1)G(b - p_B) - (n-1)(p_a - d) = 0,$$

while the first-order condition of seller $i \neq a$'s maximizing choice of p_i is

$$(n-1)t + \sum_{j \neq i}^{n} (p_j - p_i) + G(b - p_B + (n-1)p_S) - (n-1)(p_i - d) = 0.$$

Imposing the symmetry condition on all sellers $i \neq a$ and denoting the equilibrium prices p_a^* and p_i^*, for $i \neq a$, we have

$$p_a^* = t + d - \frac{n-1}{2n-1}G\beta,$$

$$p_i^* = t + d + Gp_S + \frac{1}{2n-1}G\beta.$$

The respective market shares are

$$s_{ai}^m = \frac{1}{2} + \frac{p_i^* - (b - p_B) - p_a^*}{2t} = \frac{1}{2} + \frac{Gp_S + \frac{n}{2n-1}G\beta - (b - p_B)}{2t},$$

$$s_{ai}^d = \frac{1}{2} + \frac{p_i^* - p_a^*}{2t} = \frac{1}{2} + \frac{Gp_S + \frac{n}{2n-1}G\beta}{2t},$$

$$s_{ij}^m = s_{ij}^d = \frac{1}{2},$$

where the superscript m denotes the buyers who have joined M and d denotes buyers who have not.

Denote the equilibrium profit of seller a as π_a^* and seller $i \neq a$ as π_i^*. Thus,

$$\pi_a^* = \frac{1}{n}\left(1 + \frac{p_i^* - p_a^* - (b - p_B)G}{t}\right)(p_a^* - d)$$

$$= \frac{t}{n}\left(1 - \frac{(n-1)G\beta}{(2n-1)t}\right)^2,$$

$$\pi_i^* = \frac{G}{n(n-1)}\left(\frac{p_a^* + b - p_B - p_i^*}{t} + n - 1\right)(p_i^* - p_S - d) + \frac{1-G}{n(n-1)}(n - 1 + \frac{p_a^* - p_i^*}{t})(p_i^* - d)$$

$$= \frac{1}{n}\left(1 - \frac{Gp_S + \frac{nG\beta}{2n-1}}{(n-1)t}\right)\left(t + \frac{G\beta}{2n-1}\right) + \frac{G(b - p_B)}{n(n-1)t}\left(t + Gp_S + \frac{G\beta}{2n-1} - p_S\right)$$

Clearly, π_i^* is decreasing in p_S.

For the equilibrium to exist, we need to ensure seller $i \neq a$ does not want to deviate. Suppose seller i deviates by not joining M, while all other sellers (except a) keep joining M and charge the equilibrium price p_i^* and seller a charges the equilibrium price p_a^*. The deviation profit of seller i is

$$\pi_i' = \max_{p_i'}\left\{\begin{array}{l}\left(G\frac{2}{n(n-1)}\left(\frac{1}{2} + \frac{p_a^* - p_i'}{2t} + (n - 2)\left(\frac{1}{2} + \frac{p_i^* - (b - p_B) - p_i'}{2t}\right)\right)\right. \\ \left. + (1 - G)\frac{2}{n(n-1)}\left(\frac{1}{2} + \frac{p_a^* - p_i'}{2t} + (n - 2)\left(\frac{1}{2} + \frac{p_i^* - p_i'}{2t}\right)\right)\right)(p_i' - d).\end{array}\right\}$$

Thus, the optimal deviation price is

$$p_i' = t + d - \frac{2n - 3}{2(2n - 1)}G\beta$$

and the optimal deviation profit is

$$\pi_i' = \frac{t}{n}\left(1 - \frac{(2n - 3)G\beta}{2(2n - 1)t}\right)^2.$$

Clearly, π_i' is increasing in p_S.

Thus, seller i is willing to joining M if and only if $\pi_i' \leq \pi_i^*$. We have that $\pi_i^* > \frac{t}{n} > \pi_i'$ when $p_S = 0$, and $\pi_i^* < \frac{t}{n} = \pi_i'$ when $p_S = b - p_B$. Given that π_i^* is decreasing in p_S and π_i' is increasing in p_S, the maximum fee to sellers (say $\bar{p}_S$) must ensure $\pi_i^* = \pi'$, with $0 < \bar{p}_S < b - p_B$. Also for given values of b, p_B and G, $\bar{p}_S$ is uniquely defined.

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We now turn to buyers' joining decision. The expected surplus a buyer gets from joining M is

$$CS^m = v - \frac{2}{n}\left(s_{ai}^m\left(\frac{t}{2}s_{ai}^m + p_a^*\right) + (1-s_{ai}^m)\left(\frac{t}{2}(1-s_{ai}^m) + p_i^* - b + p_B\right)\right)$$
$$- \frac{n-2}{n}\left(\frac{t}{2}s_{ij}^m + p_i^* - b + p_B\right)$$

and the expected surplus a buyer gets from not joining M is

$$CS^d = v - \frac{2}{n}\left(s_{ai}^d\left(\frac{t}{2}s_{ai}^d + p_a^*\right) + (1-s_{ai}^d)\left(\frac{t}{2}(1-s_{ai}^d) + p_i^*\right)\right) - \frac{n-2}{n}\left(\frac{t}{2}s_{ij}^d + p_i^*\right).$$

The expected gain of joining M is

$$CS^m - CS^d = -\frac{2}{n}\left((s_{ai}^m - s_{ai}^d)\left(\frac{t}{2}(s_{ai}^m + s_{ai}^d) + p_a^* - \frac{t}{2}(2 - s_{ai}^m - s_{ai}^d) - p_i^*\right) - (1-s_{ai}^m)(b - p_B)\right)$$
$$+ \frac{n-2}{n}(b - p_B)$$
$$= \left(\frac{2t - 2Gp_S - \frac{2n}{2n-1}G\beta + b - p_B}{2nt} + \frac{n-2}{n}\right)(b - p_B).$$

Given that sellers have joined M, buyers join M if and only if $c \leq \tilde{c}$, where $\tilde{c}$ is uniquely defined by

$$\tilde{c} = \left(\frac{2t - 2G(\tilde{c})p_S - \frac{2n}{2n-1}G(\tilde{c})\beta + b - p_B}{2nt} + \frac{n-2}{n}\right)(b - p_B). \qquad (38)$$

Note that $\tilde{c}$ is uniquely defined because the left-hand-side is strictly increasing in $\tilde{c}$ while the right-hand-side is strictly decreasing in $\tilde{c}$, for any given $b, p_B, p_S \geq 0$ and $b - p_B > 0$.

We can now summarize the equilibrium strategies. For any $b(k) \geq 0$, $p_B \geq 0$ and $b(k) - p_B > 0$, there exists a maximum fee $\bar{p}_S > 0$ such that when $p_S \leq \bar{p}_S$, there exists an asymmetric equilibrium where (i) all sellers join M except seller a; (ii) seller a sets a price equal to $p_a^* = t + d - \frac{n-1}{2n-1}G(\tilde{c})\beta$ and all other sellers set the price $p_i^* = t + d + G(\tilde{c})p_S + \frac{1}{2n-1}G(\tilde{c})\beta$; and (iii) buyers join M if $c \leq \tilde{c}$, where $\tilde{c}$ is uniquely defined by (38).

The demand for M can be written

$$D^m = G(\tilde{c})\left(\frac{2}{n}(1 - s_{ai}^m) + \frac{n-2}{n}\right).$$

The expressions for the consumer surplus and welfare generated by M are given by the same general expressions as in the case without price coherence.

Large number of sellers

Consider the limit as $n \to \infty$. Then $\tilde{c}$ converges to $b(k) - p_B$. The demand for M converges to $G(b(k) - p_B)$. Moreover, the value of $\bar{p}_S$ solving $\pi_i^*(\bar{p}_S) = \pi'(\bar{p}_S)$ converges to $b(k) - p_B$. The result therefore converges to the case in which all sellers join in Section 4.2, reflecting that the alternative seller becomes irrelevant to buyers' decisions because the chance that a buyer ends up buying from

the alternative seller approaches zero.

G.3 Numerical results

We use a numerical example to illustrate what happens for a finite number of sellers. Assume $b(k) = \sqrt{k}$. Assume buyers' joining costs are drawn from a uniform distribution, so $G(c) = c$, where $c \in [0, 1]$. Assume that $t = 10$ and $n = 5$.

Table OA-1 shows the investment level, seller fee, the number of buyers joining M (which equals $\tilde{c}$), M's profit, and the effect of M's presence on sellers' aggregate profit, consumer surplus and total welfare, both without price coherence and with price coherence. We omit the buyer fee p_B because it is normalized to zero in the case without price coherence and is optimally set at zero in the case with price coherence.

Table OA-1: A numerical example with $n = 5$, $t = 10$

	k	p_S	$\tilde{c}$	Π_M	$\Sigma_i \Delta \pi_i$	ΔCS	ΔW
Without price coherence	0.250	0.375	0.080	0.008	0.000	0.005	0.013
With price coherence	0.436	0.656	0.528	0.093	-0.002	-0.136	-0.045

Clearly, M prefers to impose price coherence. Doing so gives it 52.8% of buyers rather than 8% of buyers without price coherence, and increases its profit more than eleven-fold. Given that the efficient amount of investment with this number of sellers is $k^e = 0.25$, M optimally invests without price coherence and over-invests with price coherence. A social planner would optimally set $k = k^e = 0.25$, $p_B = k^e$ and $p_S = 0$, which would result in the efficient number of buyers joining M, with $\tilde{c}^e = 0.160$.[16] Too few buyers join without price coherence and too many buyers join with price coherence.[17] The contribution of M to consumer surplus is positive without price coherence and negative with price coherence. Thus, Proposition 3 in the main text holds for this example. Additionally, with price coherence, M contributes negatively to sellers' aggregate profits and welfare.

Using the same parameter values, Table OA-2 considers how a change in the number of sellers n affects these results. As shown analytically in Sections G.1-G.2, the case studied in Section 4 in the main text corresponds to the case in which $n \to \infty$. For all values of n considered, the qualitative results of Proposition 3 continue to hold, as do the additional results noted in Section 6.4. As the table shows, the distortions created by price coherence increase as the number of sellers increases, reflecting that the alternative seller becomes relatively less important in buyers' decisions. (In the limit, the

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[16]Here, $\tilde{c}^e$ denotes the marginal buyer's joining cost at the efficient outcome.

[17]Given the demand from buyers to join is approximately linear in the seller fee without price coherence, the fact only about half as many buyers join compared to the efficient level reflects that M is a monopolist.

alternative seller has zero weight.) Note that the efficient investment $k^e = 0.25$ is independent of n, but the efficient number of buyers joining M ($\tilde{c}^e$) increases in n as the proportion of sellers offering benefits $b(k)$ to buyers increases. Nevertheless, there are still too few buyers joining M without price coherence and too many buyers joining M with price coherence.

Table OA-2: Results for varying number of sellers n, without and with price coherence

n	$\tilde{c}^e$	without price coherence							with price coherence						
		k	p_S	$\tilde{c}$	Π_M	$\Sigma_i\Delta\pi_i$	ΔCS	ΔW	k	p_S	$\tilde{c}$	Π_M	$\Sigma_i\Delta\pi_i$	ΔCS	ΔW
2	0.063	0.250	0.374	0.031	0.002	0.000	0.001	0.003	0.402	0.608	0.321	0.034	-0.004	-0.045	-0.015
3	0.112	0.250	0.375	0.056	0.005	0.000	0.003	0.008	0.426	0.640	0.436	0.064	-0.003	-0.090	-0.030
4	0.141	0.250	0.375	0.071	0.007	0.000	0.004	0.011	0.433	0.651	0.494	0.082	-0.003	-0.118	-0.039
5	0.160	0.250	0.375	0.080	0.008	0.000	0.005	0.013	0.436	0.656	0.528	0.093	-0.002	-0.136	-0.045
10	0.203	0.250	0.375	0.101	0.011	0.000	0.006	0.018	0.441	0.662	0.598	0.119	-0.001	-0.177	-0.059
50	0.240	0.250	0.375	0.120	0.015	0.000	0.008	0.022	0.444	0.666	0.653	0.142	0.000	-0.213	-0.071
100	0.245	0.250	0.375	0.123	0.015	0.000	0.008	0.023	0.444	0.666	0.660	0.145	0.000	-0.217	-0.072
500	0.249	0.250	0.375	0.125	0.016	0.000	0.008	0.023	0.444	0.667	0.665	0.148	0.000	-0.221	-0.074
∞	0.250	0.250	0.375	0.125	0.016	0.000	0.008	0.023	0.444	0.667	0.667	0.148	0.000	-0.222	-0.074

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